SOLUTIONS (Sample Paper – 1)

1. **(a)** Given equations are 3x + 4y = 5 ⇒ 3x + 4y – 5 = 0 and 6x + 8y = 7 ⇒ 6x + 8y – 7 = 0

Here, $a_1 = 3$, $b_1 = 4$, $c_1 = -5$ and $a_2 = 6$, $b_2 = 8$, $c_2 = -7$

Now, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ Given, pair of straight lines are parallel

2. **(a)** We have, x = a tan θ and y = b sec θ

$\Rightarrow \tan\theta = \frac{x}{a}$ and $\sec\theta = \frac{y}{b}$

Putting these values in $\sec^2\theta - \tan^2\theta = 1$, we get

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

3. **(a)** Total number of outcomes = 6

Favourable number of outcomes {2, 4, 6} = 3

∴ Required probability = $\frac{3}{6} = \frac{1}{2}$

4. **(a)** We have, $p(x) = 2x^2 - 3x - 9$

$\Rightarrow p(x) = 2x^2 - 6x + 3x - 9$

$= 2x(x - 3) + 3(x - 3)$

$= (x - 3)(2x + 3)$

For zeros of the polynomial p(x),

Put p(x) = 0

⇒ (x – 3) (2x + 3) = 0

⇒ (x – 3) = 0 and (2x + 3) = 0

⇒ x = 3, –3/2

5. **(c)** Let A(a + b, a – b), B (2a + b, 2a – b),

C(a – b, a + b) and D(x, y) be the vertices of a parallelogram

The mid-points of the diagonals BD and AC have the same co-ordinates.

$\therefore \left[\frac{2a+b+x}{2}, \frac{2a-b+y}{2}\right] = \left[\frac{a+b+a-b}{2}, \frac{a-b+a+b}{2}\right]$

⇒ 2a + b + x = 2a and 2a – b + y = 2a

⇒ x = – b and y = b

Hence, the fourth vertex is D(– b, b)

6. **(a)** From the figure, we have

$\sin\theta = \frac{PQ}{PR} = \frac{8}{10} = \frac{4}{5}; \cos\theta = \frac{QR}{PR} = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{PQ}{PR} = \frac{8}{6} = \frac{4}{3}$

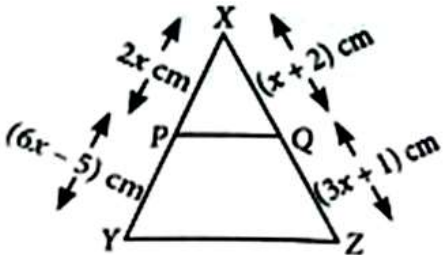
$\therefore 25(\sin^2\theta + 2\cos^2\theta - \tan\theta) = 25\left(\frac{16}{25} + 2\times\frac{9}{25} - \frac{4}{3}\right)$

$= 25\left(\frac{48+54-100}{25\times 3}\right) = \frac{2}{3}$

7. **(d)** We have, a = – 10, d = –6 + 10 = 4

$\therefore$ a_{16} = a + 15d = –10 + 15 × 4 = 50

8. **(a)** In ΔXYZ, PQ || YZ



$\therefore \quad \frac{XP}{PY} = \frac{XQ}{QZ}$

(By basic proportionality theorem)

$\Rightarrow \frac{2x}{6x-5} = \frac{x+2}{3x+1}$

$\Rightarrow$ (2x) (3x + 1) = (x + 2) (6x – 5)

$\Rightarrow 6x^2 + 2x = 6x^2 + 12x - 5x - 10$

$\Rightarrow$ 5x = 10 $\Rightarrow$ x = 2

9. **(c)** We known that, Mode = 3 Median – 2 Mean

$\Rightarrow$ 3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median = 16 + 2 × 28 $\Rightarrow$ Median = 72/3 = 24

10. **(a)** If we extend the lines. They intersect each other at a unique point, which is the solution of the pair of linear equations. Hence, the lines are consistent with unique solution.

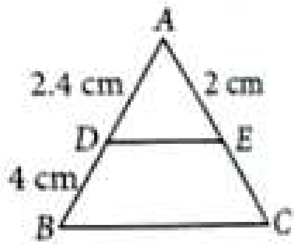
11. **(a)** Given points are P(36, 15) and O(0, 0)

$\therefore$ Distance between the points,

$OP = \sqrt{(0-36)^2 + (0-15)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$ units

12. **(c)** In ΔABC, DE || BC

By Basic Proportionality Theorem,



$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{2.4}{4} = \frac{2}{EC} \Rightarrow EC = \frac{2 \times 4 \times 40}{24}$

$\Rightarrow \quad EC = \frac{10}{3}$

$\therefore$ AC = AE + EC = 2 + $\frac{10}{3} = \frac{16}{3}$ cm

13. **(c)** Let E be the event of getting a number on the die which is not a factor of 36 and this number is 5.

Total number of possible outcomes = 6

Number of outcomes favourable to event E = 1

$\therefore$ Required probability = $\frac{1}{6}$

14. **(c)** Total number of possible outcomes = 100

Favourable number of outcomes = 75

P(Selecting a defective memory card) = $\frac{75}{100} = \frac{3}{4}$

15. **(d)** Smallest prime number = 2

Smallest old composite number = 9

$\therefore$ LCM (2, 9) = 18

16. **(b)** Here, y = p(x) touches the x-axis at one point

So, number of zeroes is one

17. **(a)** Let the required ratio be k: 1

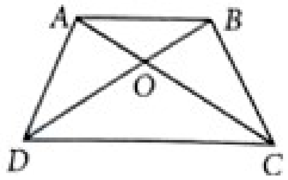
$\therefore \dfrac{(k)(4)+(1)(-3)}{k+1} = -2, \dfrac{(k)(-9)+(1)\quad(5)}{k+1} = 3$

$\Rightarrow \dfrac{4k-3}{k+1} = -2, \dfrac{-9k+5}{k+1} = 3$

$\Rightarrow$ 4k – 3 = – 2k – 2, –9k + 5 = 3k + 3

$\Rightarrow$ 6k = 1, 12k = 2 $\Rightarrow$ $k = \dfrac{1}{6}, k = \dfrac{1}{6}$

18. **(d)**



$\dfrac{AO}{OC} = \dfrac{BO}{OD}$ (∵ Given)

$\angle$AOB = $\angle$DOC (vertically opposite angles)

$\Rightarrow$ ΔAOB ~ ΔCOD (By SAS criterion)

$\Rightarrow$ $\angle$ABO = $\angle$CDO

These are alternate angles.

$\Rightarrow$ AB || CD

$\therefore$ ABCD is a trapezium

19. **(a)** We have $5x^2 + 14x + 10 = 0$

$\therefore$ D = $(14)^2$ – 4.5.10 = 196 – 200 = – 4 < 0

As D < 0, no real roots.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. **(c)** Clearly, reason is false.

Given, sin $A = \dfrac{8}{17} \cos A = \sqrt{1-\left(\dfrac{8}{17}\right)^2} = \sqrt{\dfrac{289-64}{17^2}} = \dfrac{15}{17}$

$\tan A = \dfrac{\sin A}{\cos A} = \dfrac{8/17}{15/17} = \dfrac{8}{15}$

$\therefore$ Assertion (A) is true but reason (R) is false.

21. ∵ BD bisects both $\angle$B and $\angle$D

Then, $\angle$ADB = $\angle$CDB …(i)

and $\angle$ABD = $\angle$CBD …(ii)

(i) In, ΔABD and ΔCBD, we have

$\angle$ADB = $\angle$CDB (using (i))

$\angle$ABD = $\angle$CDB (using (ii))

$\therefore$ ΔABD ~ ΔCBD (By AA criteria)

(ii) As, ΔABD ~ ΔCBD

$\therefore$ AB = BC (By corresponding parts of similar triangles)

22. **(a)** We have,

$$\text{L.H.S.} = \frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \frac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}}{\sin\theta\cos\theta} = \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$= \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta} = \sec^2\theta - \operatorname{cosec}^2\theta = \text{R.H.S}$

OR

(b) Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 2\sin^2\theta - (1 - \sin^2\theta) = 2$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow 2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow 2\sin^2\theta = 3 \Rightarrow \sin 2\theta = 1$

$\Rightarrow \sin\theta = 1 = \sin 90^\circ$ [$\because \sin 90^\circ = 1$]

$\therefore \theta = 90^\circ$

23. Here, $OA \perp PA$ and $OB \perp PB$

[$\because$ Tangent at any point of a circle is perpendicular to the radius through the point of contact]

In ΔPAO, $OP_2 = AP^2 + OA^2 = 15^2 + 8^2 = 225 + 64 = 289$

$\Rightarrow OP = 17$ cm

In ΔPBO, $PB^2 = OP^2 - OB^2 = 172 - 7^2 = 289 - 49 = 240$

$\Rightarrow PB = \sqrt{240}\text{ cm} = 4\sqrt{15}\text{ cm}$

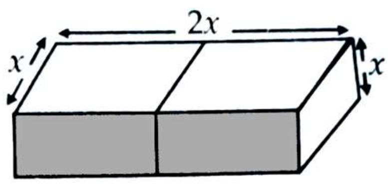
24. Let the side of each cube be x.

Given, volume of each cube

= 125 cm^3

$\therefore x^3 = 125\ cm^3$

$\Rightarrow x = 5$ cm



Now, length of resulting cuboid (*l*) = 2x = 10 cm

Breadth of resulting cuboid (b) = x = 5 cm

Height of resulting cuboid (h) = x = 5 cm

$\therefore$ Surface area of the cuboid = 2 (*l*b + bh + hl)

= 2(10 × 5 + 5 × 5 + 5 × 10) = 2 [50 + 25 + 50] = 250 cm^2

25. **(a)** We have, $(k + 4)x^2 + (k + 1)x + 1 = 0$

Here, a = k + 4, b = k + 1 and c = 1

$\therefore D = b^2 - 4ac = (k + 1)^2 - 4(k + 4)(1)$

$= k^2 + 1 + 2k - 4k - 16 = k^2 - 2k - 15$

$k^2 - 5k + 3k - 15 = k(k - 5) + 3(k - 5) = (k - 5)(k + 3)$

Now, the roots of the given equation are real and equal.

$\therefore D = 0 \Rightarrow (k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$

OR

(b) We have, $x^2 - 3\sqrt{11}x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1,\ b = -3\sqrt{11}$ and $c = 3$

$D = b^2 - 4ac = (-3\sqrt{11})^2 - 4(1)(3) = 99 - 12 = 87 > 0$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-3\sqrt{11}) \pm \sqrt{87}}{2(1)} = \dfrac{3\sqrt{11} \pm \sqrt{87}}{2}$

Taking positive sign, $x = \dfrac{3\sqrt{11} + \sqrt{87}}{2}$

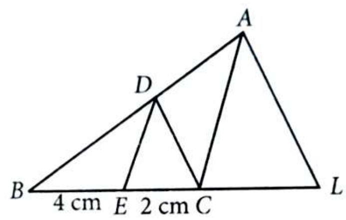
Taking negative sign, $x = \dfrac{3\sqrt{11} - \sqrt{87}}{2}$

26. Let CL = x cm

In ΔABC, DE || AC

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$

[By basic proportionality theorem]



$\Rightarrow \frac{BD}{DA} = \frac{4}{2} \Rightarrow \frac{BD}{DA} = 2$ …(i)

In ΔBLA, CD || LA

$\Rightarrow \frac{BC}{CL} = \frac{BD}{DA}$ [By basic proportionally theorem]

$\Rightarrow \frac{4+2}{x} = \frac{BC}{DA} \Rightarrow \frac{6}{x} = \frac{BD}{DA}$ …(ii)

From (i) and (ii), we have $\frac{6}{x} = 2 \Rightarrow x = 3$

Hence, CL = 3 cm

27. **(a)** Given $\frac{a_{10}}{a_{30}} = \frac{1}{3}$ and $S_6 = 42$

As $a_n = a + (n - 1)d$, where a is the first term, d is the common difference, n is number of terms and a_n is n^{th} term of given A.P.

$\therefore \frac{a+9d}{a+29d} = \frac{1}{3}$

$\Rightarrow 3a + 27d = a + 29d \Rightarrow 2a = 2d \Rightarrow a = d$ …(i)

Now S6 = 42

$\Rightarrow \frac{6}{2}[2 \times a + (6-1)d] = 42 \quad \left[s_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow 3(2a + 5d) = 42$

$\Rightarrow 2a + 5a = 14$

$\Rightarrow 7a = 14 \Rightarrow a = 2$ [using (i)]

Hence, a = d = 2

First term and the common difference of given A.P. are 2, 2.

OR

(b) Let n and d be number of terms and common difference of the A.P. respectively.

We have, first term (a) = 5; last term (l) = a_n = 45

Sum of all terms (S_n) = 400

$\Rightarrow \frac{n}{2}(a + l) = 400 \Rightarrow \frac{n}{2}(5 + 45) = 400 \Rightarrow n = \frac{800}{50} = 16$

Now, an = a + (n – 1) d

$\Rightarrow 45 = 5 + (16 - 1)d$

$\Rightarrow 40 = 15d \Rightarrow d = \frac{40}{15} = \frac{8}{3}$

28. Here, mode is 48, which lies in the interval 40-50

$\therefore$ Modal class is 40-50

So, $l = 40$, $f_0 = 12$, $f_1 = p$, $f_2 = 18$, $h = 10$

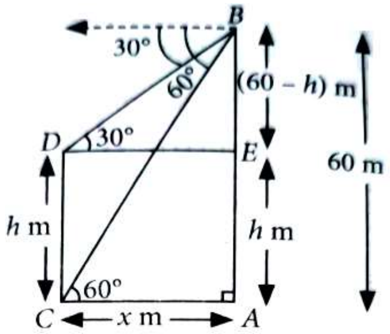
$$\therefore \text{ Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow 48 = 40 + \left(\frac{p-12}{2p-12-18}\right)\times 10 \Rightarrow 8 = \frac{10p-120}{2p-30}$$

$\Rightarrow 16p - 240 = 10p - 120 \Rightarrow 6p = 120 \Rightarrow p = 20$

29. Let AB be the building and CD be the tower of height h metres. It is given that, $\angle EDB = 30^\circ$ and $\angle ACB = 60^\circ$ Let AC = DE = x metres

In right $\triangle DEB$,



$$\tan 30^\circ = \frac{BE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$$

$\Rightarrow x = \sqrt{3}(60 - h)$ ...(i)

In right $\triangle CAB$, $\tan 60^\circ = \frac{AB}{CA}$

$$\Rightarrow \sqrt{3} = \frac{60}{x} \Rightarrow x = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Putting the value of x in (i), we get

$$20\sqrt{3} = \sqrt{3}(60-h) \Rightarrow 20 = 60 - h$$

$\Rightarrow h = 60 - 20 = 40$ metres

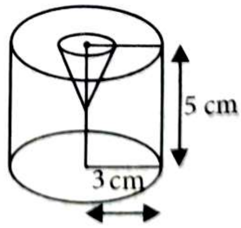
Thus, the required height of the tower is 40 m.

30. Radius of cylinder, (r) = 3 cm

Height of cylinder, (h) = 5 cm

$\therefore$ Volume of cylinder

$= \pi r^2 h = \pi(3)^2 \times 5 = 45\pi \text{ cm}^3$



Radius of cone (r_1) = $\frac{3}{2}$ cm

Height of cone (h_1) = $\frac{8}{9}$ cm

Volume of cone = Volume of metal taken out

$$= \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi \text{ cm}^3$$

Volume of metal left in the cylinder

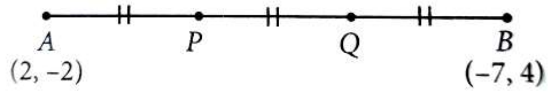
= Volume of Cylinder – Volume of cone

$$= 45\pi - \frac{2}{3}\pi = \frac{135\pi - 2\pi}{3} = \frac{133\pi}{3}\text{cm}^3$$

$\therefore$ Required ratio

$$= \frac{\text{Volume of metal left in cylinder}}{\text{Volume of metal taken out}} = \frac{\frac{133\pi}{3}}{\frac{2}{3}\pi} = \frac{133}{2} = 133 : 2$$

31. **(a)** Let $p(x_1, y_1)$ and $Q(x_2, y_2)$ divides AB into three equal parts.



$\therefore$ P divides AB in the ratio 1: 2

Using section formula, we have

$$(x_1, y_1) = \left(\frac{-7+4}{1+2}, \frac{4-4}{1+2}\right) = \left(\frac{-3}{3}, 0\right) = (-1, 0)$$

$\therefore$ Coordinates of P are (–1, 0)

Also, Q divides AB in the ratio 2: 1

Using section formula, we have

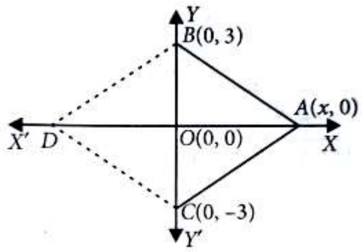
$$(x_2, y_2) = \left(\frac{-14+2}{2+1}, \frac{8-2}{2+1}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

$\therefore$ Coordinates of Q are (–4, 2

OR

(b) Q is the mid-point of the base BC.

$\therefore$ Coordinates of point B are (–0, 3)



So, BC = $\sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{6^2} = 6$ units.

Let the coordinates of point A be (x, 0)

$\therefore$ $AB = \sqrt{(0-x)^2 + (3-0)^2}$

$= \sqrt{x^2 + 9}$

Also, AB = BC (∵ ΔABC is an equilateral triangle)

$\therefore$ Coordinates of point A = (x, 0) = $(-3\sqrt{3}, 0)$

32. Suppose that c = 0, then $a + bn^{1/3} + cn^{2/3} = 0$ …(1)

$\Rightarrow$ $a + bn^{1/3} = 0$

But $n^{1/3}$ is irrational as n is not a perfect cube of a rational number, therefore we obtain a = b = 0

Thus, a = b = c = 0

If $c \neq 0$, then on multiplying both sides of (1) by $n^{1/3}$, we get

$an^{1/3} = bn^{2/3} + cn = 0$ …(2)

Multiplying (1) by b and (2) by c and then subtracting, we get

$(ab + b^2 n^{1/3} + bcn^{2/3}) - (acn^{1/3} + bcn^{2/3} + c^2n) = 0$

$\Rightarrow$ $(b^2 - ac)n^{1/3} + (ab - c^2n) = 0$ (3)

$\Rightarrow$ $b^2 - ac = 0$ and $ab - c^2n = 0$

(∵ $n^{1/3}$ is an irrational number as n is not a perfect cube)

$\Rightarrow b^2 = ac$ and $ab = c^2n \Rightarrow b^2 = ac$ and $a^2b^2 = c^4n^2$

$\Rightarrow$ $a^2(ac) = c^4n^2$

$\Rightarrow$ $n^2 = \frac{a^3}{c^3} = \left(\frac{a}{c}\right)^3$

$\frac{a^3}{c^3} = \left(\frac{a}{c}\right)^3$ n^2 is a perfect cube of a rational number, which is a contradiction.

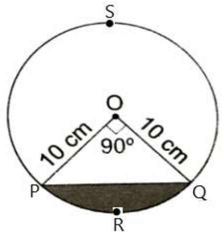
(∵ If n is not a perfect cube, then n^2 also not n perfect cube)

Thus, $c \neq 0$ is not possible and we must have a = b = c = 0

33. **(a)** We have, radius (r) = 10 cm and θ = 90°

Area of minor segment PQR = Area of sector OPRQ – Area of ΔOPQ

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} \times OP \times OQ$$



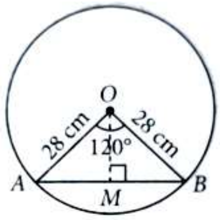
$$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$$

$$= \frac{550}{7} - 50 = 28.5\,cm^2$$

Area of major segment PSQ = Area of circle – Area of minor segment

= $\pi(10)^2$ – 28.5 = 314 – 28.5 = 085.5 cm^2

OR



(b) Let O be the centre of circle and AB be the chord

(i) Length of are $\widehat{AB} = \frac{\theta}{360^\circ} \times 2\pi r$

$$= \frac{120^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 28 = \frac{1}{3} \times 2 \times 22 \times 4 = 58.67\,cm^2$$

(ii) Area of minor sector, $AOBA = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28 = \frac{1}{3} \times 22 \times 4 \times 28 = 821.33\,cm^2$$

Area of major sector = πr^2 – Area of minor sector

$$= \frac{22}{7} \times 28 \times 28 - 821.33 = 2464 - 821.33 = 1642.67\,cm^2$$

(iii) Draw OM ⊥ AB

Then, OM is the perpendicular bisector of AB and also bisects ∠AOB

In ΔAOM, $\frac{AM}{OA}$ = sin 60° and $\frac{OM}{OA}$ = cos 60°

$\Rightarrow \frac{AM}{28} = \frac{\sqrt{3}}{2}$ and $\frac{OM}{28} = \frac{1}{2}$ $\Rightarrow$ AM = $\frac{28\sqrt{3}}{2} = 14\sqrt{3}$ cm and OM = 14 cm

AB = 2 × AM = 2 × $14\sqrt{3} = 28\sqrt{3}$

∴ Area of ΔAOB $= \frac{1}{2} \times OM \times AB = \frac{1}{2} \times 14 \times 28\sqrt{3}$ = 196 × 1.7 = 333.2 cm^2

So, Area of segment = Area of sector AOBA

– Area of ΔAOB = (821.33 – 333.2) cm^2 = 488.13 cm^2

34. Since, length of the tangents from an external point to a circle are equal.

∴ AF = AE [Tangents from A] …(i)

BD = BF [Tangents from B] …(ii)

and CE = CD [Tangents from C] …(iii)

Adding equation (i), (ii) and (iii), we get

AF + BD + CE = AE + BF + CD …(iv)

Now, perimeter of ΔABC = AB + BC + AC

= (AF + FB) + (BD + CD) + (AE + EC)

= (AF + AE) + (BF + BD) + (CD + CE)

= 2AF + 2BD + 2CE [Using (i), (ii) and (iii)]

= 2(AF + BD + CE)

$\Rightarrow$ AF + BD + CE = $\frac{1}{2}$ (Perimeter of ΔABC) …(v)

Hence, AF + BD + CE = AE + BF + CD

= $\frac{1}{2}$ (Perimeter of ΔABC) [From (iv) and (v)]

35. **(a)** We have, (2p + 2q)x – 4qy – (7p + 4q + 3) = 0
and 5x – y – 16 = 0
Here, a_1 = 2p + 2q, b_1 = –4q, c_1 = – (7p + 4q + 3)
and a_2 = 5, b_2 = –1, c_2 = – 16
For infinitely many solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2p+2q}{5} = \frac{-4q}{-1} = \frac{-(7p+4q+3)}{-16}$$

Taking first and last terms, we get

$$\frac{2p+2q}{5} = \frac{7p+4q+3}{16}$$

$\Rightarrow$ 32p + 32q = 35p + 20q + 15
$\Rightarrow$ 32q – 20q = 35p – 32p + 15
$\Rightarrow$ 12q = 3p + 15 $\Rightarrow$ p = 4q – 5 …(i)
Taking last two terms, we get
4q × 16 = 7p + 4q + 3
$\Rightarrow$ 64q = 7p + 4q + 3 $\Rightarrow$ 7p – 60q = – 3 …(ii)
Using (i) in (ii), we get 7(4q – 5) – 60q = – 3
28q – 35 – 60q = – 3 $\Rightarrow$ –32q = 32 $\Rightarrow$ q = – 1
Using q = – 1 in (i), we get, p = 4(–1) –5 = –9

OR

(b) Let Ravi invested Rs x at 8% simple interest and Rs y at 9% simple interest.

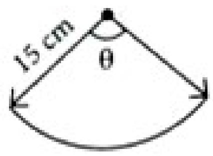
According to the first condition, $\frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} = 163$

$\Rightarrow$ 8x + 9y = 16300 …(i)
According to the second condition,

$$\frac{x \times 9 \times 1}{100} + \frac{y \times 8 \times 1}{100} = 160$$

$\Rightarrow$ 9x + 8y = 16000 …(ii)
Multiplying (i) by 9 and (ii) by 8, we get
72x + 81y = 146700 …(iii)
and 72x + 64y = 128000 …(iv)
Subtracting (iv) from (iii), we get
17y = 18700 $\Rightarrow$ y = 1100
Putting y = 1100 in (i), we get 8x + 9(1100) = 16300
$\Rightarrow$ 8x = 16300 – 9900 = 6400 $\Rightarrow$ x = 800
Hence, he invested Rs 800 at 8% simple interest and Rs 1100 at 9% simple interest.

36. (i) we have, r = 15 cm



l = $\frac{1}{2}$(44) = 22 cm

we known that l = $2\pi r\left(\frac{\theta}{360°}\right)$

$$\Rightarrow \ \theta = \frac{22 \times 360°}{2 \times \frac{22}{7} \times 15} = \frac{180° \times 7}{15} = 12° \times 7 = 84°$$

(ii) Angle made by hour hand in 12 hours = 360°

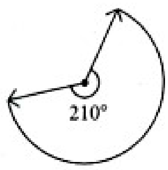
Angle made by hour in 30 minutes $= \left(\frac{360°}{12} \times \frac{1}{2}\right) = 15°$

(iii) (a) Angle made by minute hand in 60 minutes = 360°
Angle made by minute hand in 35 minutes

$$= \frac{360°}{60} \times 35 = 210°$$

Length of minute hand = 10 cm
$\therefore$ Area swept by minute hand in
35 minutes = Area of sector of an angle 210°

$$= \pi r^2 \left(\frac{210°}{360°}\right) = \frac{22}{7} \times 10 \times 10 \times \frac{7}{12} = \frac{2200}{12} = 183.33\,\text{cm}^2$$



OR

(iii) (b) Angle made by hour hand in 1 hour = $\frac{360°}{12} = 30°$

Also, r = 7 cm
$\therefore$ Area swept by hour hand in 1 hour = Area of sector of an angle 30°

$$= \pi r^2 \times \left(\frac{30°}{360°}\right) = \frac{22}{7} \times 7 \times 7 \times \frac{1}{12} = \frac{154}{12} = 12.83\,\text{cm}^2$$

Number of hours from 9 a.m. to 5 p.m. = 8
Area swept by hour hand in 1 hour = 12.83 cm^2
$\therefore$ Area swept by hour hand in 8 hours = 12.83 × 8 = 102.64 cm^2

37. Number of cakes in 1^{st}, 2^{nd}, 3^{rd} row, ... are 3, 5, 7, …
So, it forms an A.P. with first term a = 3, d = 5 – 3 = 2
(i) Cakes in 17^{th} row = t_{17} = 3 + 16(2) = 35
cakes in 10^{th} row = t_{10} = 3 + 9(2) = 21
$\therefore$ Required difference = 35 – 21 = 14
(ii) Let n be the number of rows required
$\therefore S_n = 120$

$$\Rightarrow \ \frac{n}{2}[2(3) + (n-1)2] = 120$$

$\Rightarrow$ $n^2 + 2n - 120 = 0 \Rightarrow n^2 + 12n - 10n - 120 = 0$
$\Rightarrow$ $(n + 12)(n - 10) = 0 \Rightarrow n = 10$
So, 10 rows are required to put 120 cakes.
(iii) (a) No. of cakes in 5^{th} row = t_5 = 3 + 4(2) = 11
No. of cakes in 8^{th} row = t_8 = 3 + 7(2) = 17
$\therefore$ Required sum = 11 + 17 = 28
No. of cakes in 30^{th} row = t30 = 3 + 29(2) = 61

OR

(iii) (b) Here n = 15
$\therefore$ t_{15} = 3 + 14(2) = 3 + 28 = 31

38. (i) In ΔABJ and ΔADH, $\angle B = \angle D = 90°$

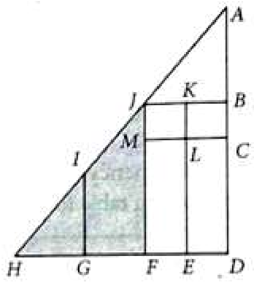
$\angle A = \angle A$ (common)

$\therefore$ By AA similarity criterion, $\Delta ABJ \sim \Delta ADH$

(ii) In ΔADH and ΔJFH, $\angle D = \angle F = 90°$

$\angle H = \angle H$ (common)

$\therefore$ By AA similarity criteria, $\Delta ADH \sim \Delta JFH$



$$\therefore \frac{AD}{JF} = \frac{AH}{JH} \Rightarrow \frac{12}{JM + MF} = \frac{8}{4} \Rightarrow \frac{12 \times 4}{8} = JM + 4$$

$\Rightarrow$ 6 = JM + 4 $\Rightarrow$ JM = 2m

(iii) (a) In ΔADH, IG || AD

$$\therefore \frac{IH}{AI} = \frac{HG}{GD}$$ (By Thales theorem)

$$\Rightarrow \frac{y+3}{2y} = \frac{y}{2y-1}$$

$\Rightarrow$ $(y + 3)(2y - 1) = y(2y) \Rightarrow 2y^2 - y + 6y - 3 = 2y^2$

$\Rightarrow 5y - 3 = 0 \Rightarrow y = 3/5$

OR

(iii) (b) Since, $\Delta ABJ \sim \Delta ADH$ (By AA similarity criterion]

$$\therefore \frac{AB}{AD} = \frac{AJ}{AH} \Rightarrow \frac{AB}{AJ} = \frac{5}{7}$$

SOLUTIONS (Sample Paper – 2)

1. **(a)** We have, p(x) = x_2 + x – 1, $\alpha + \beta = -1$ and $\alpha . \beta = -1$

Now, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-1}{-1}=1$

2. **(d)** Let the coordinates of B be (α, β). So, the coordinates of C are

$$\left(\frac{3\alpha+4\times2}{3+4},\frac{3\beta+4\times5}{3+4}\right)=\left(\frac{3\alpha+8}{7},\frac{3\beta+20}{7}\right)$$

But, the coordinates of C are (–1, 2). [Given]

$\therefore \quad \frac{3\alpha+8}{7}=-1$ and $\frac{3\beta+20}{7}=2$

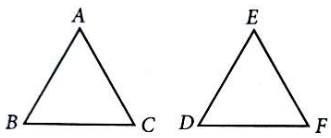
$\Rightarrow \quad \alpha = -5$ and $\beta = -2$

Thus, the coordinates of B are (–5, –2)

3. **(c)** Given, in ΔABC and ΔEDF,

$\frac{AB}{DE}=\frac{BC}{FD}$

So, $\Delta ABC \sim \Delta EDF$

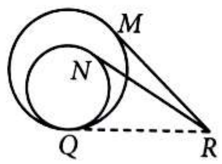


If $\angle B = \angle D$

(By SAS similarity criterion)

4. **(a)** Join RQ

Since tangents drawn from an external point to a circle are equal in length.



$\therefore$ RQ = RN ….(i) and RQ = RM ..(ii)

$\Rightarrow$ RN = RM [From (i) and (ii)]

5. **(a)** Let radius of the sphere be r cm.

According to question, $\frac{4}{3}\pi r^3 = 36\pi$

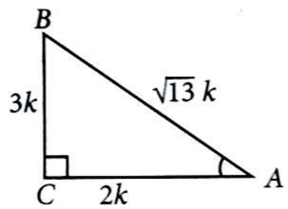
$\Rightarrow r^3 = \frac{3\times36}{4} = 27 \Rightarrow \pi = 3\,\text{cm}$

6. **(a)** Given equations of lines are ax + by – c = 0 and lx + my – n = 0

Since, am $\neq$ bl $\Rightarrow \frac{a}{l} \neq \frac{b}{m}$

$\therefore$ The pair of equations has a unique solution

7. **(c)** We have, 2 tan A = 3



$\Rightarrow \quad \tan A = \frac{3}{2} = \frac{P}{B}$

Let P = 3k and B = 2k

$AB = \sqrt{(2k)^2 + (3k)^2}$

$= \sqrt{4k^2 + 9k^2} = \sqrt{13}\ k$ (By Pythagoras theorem)

$\Rightarrow \quad H = \sqrt{13}\ k$

$\therefore \quad \sin A = \frac{P}{H} = \frac{3}{\sqrt{13}}, \cos A = \frac{B}{H} = \frac{2}{\sqrt{13}}$

Now, $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A} = \frac{4\left(\frac{3}{\sqrt{13}}\right) + 3\left(\frac{2}{\sqrt{13}}\right)}{4\left(\frac{3}{\sqrt{13}}\right) - 3\left(\frac{2}{\sqrt{13}}\right)} = 3$

8. **(d)** Sum of the numbers 1, 2, 3, …., $n = \frac{n(n+1)}{2}$

$\therefore$ Mean = $\frac{n(n+1)}{2} \div n = \frac{n+1}{2}$

9. **(c)** Since P(b, – 4) is the mid point of line joining A(6, 6) and B(–2, 3).

$\therefore \quad \frac{6+(-2)}{2} = b \Rightarrow b = 2$

10. **(b)** Total possible outcomes are {1, 2, 3, 4, 5, 6} i.e., 6 in number
Favourable outcomes are {3, 5} i.e., 2 in number

$\therefore$ Required probability $\frac{2}{6} = \frac{1}{3}$

11. **(b)** Prime numbers between 1 to 20 are {2, 3, 5, 7, 11, 13, 17, 19}
So, total number of outcomes = 8
Favourable outcome is {13} i.e., 1

$\therefore$ Required probability = $\frac{1}{8}$

12. **(b)** We have, 8 = 2 × 2 × 2,
9 = 3 × 3, 25 = 5 × 5
HCF (8, 9, 25) = 1 and LCM (8, 9, 25) = $2^3 \times 3^2 \times 5^2$ = 1800
$\therefore$ HCF (8, 9, 25) × LCM (8, 9, 25) = 1 × 1800 = 1800

13. **(a)** Given, $\sin\alpha = \frac{\sqrt{3}}{2}$

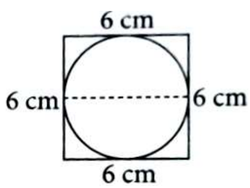
$\Rightarrow \quad \alpha = 60°$

and $\tan\beta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \beta = 30°$

Now, $\cos(\alpha - \beta) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

14. **(d)** Diameter of circle side of square
= 6 cm, then radius (r) = 3 cm
Area of circle, A = πr^2
= $\pi (3)^2 = 9\pi$ cm^2



15. **(a)** Let the numbers be a – d, a and a + d
Given, a – d + a + a + d = 18
$\Rightarrow$ 3a = 18 $\Rightarrow$ a = 6
$\therefore$ Middle term = 6

16. **(b)**

17. **(a)** Given, OB = 5 cm

Since, $\angle OBT = 90^\circ$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

So, in ΔOBT, $OT^2 = OB^2 + BT^2 = 25 + 16 = 41$ [By Pythagoras theorem]

$\Rightarrow OT = \sqrt{41}$ cm

18. **(c)** Total surface area of sold = Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

19. **(a)** Clearly, reason (R) is true

Let E be the event 'Sania win the match'.

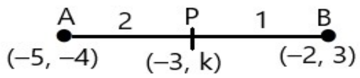
So, probability of Sania winning the match = P(E) = 0.68

$\because$ $P(E) + P(\bar{E}) = 1$

$\therefore$ Probability of Deepika winning the match = $P(\bar{E}) = 1 - 0.68 = 0.32$

$\therefore$ Assertion (A) is true and is the correct explanation of reason (R)

20. **(a)** Let P(– 3, k) divides the line segment joining the points A(–5, – 4) and B(– 2, 3) in the ratio 2: 1



Using section formula, we have

$$(-3,k) = \left(\frac{2(-2)+1(-5)}{2+1}, \frac{2(3)+1(-4)}{2+1}\right)$$

$$\Rightarrow (-3,k) = \left(\frac{-4-5}{3}, \frac{6-4}{3}\right) \Rightarrow (-3,k) = \left(-3, \frac{2}{3}\right) \Rightarrow k = \frac{2}{3}$$

Hence, both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

21. **(a)** Since, the height h is measured vertically, so $\angle EDA$ is a right angle

In ΔADE and ΔABC, $\angle ADE = \angle ABC = 90^\circ$

$\angle DAE = \angle BAC$ [Common]

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$\therefore$ $\frac{DE}{BC} = \frac{AD}{AB}$ [Since, corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{h}{8} = \frac{40}{16} \Rightarrow h = \frac{40 \times 8}{16} = 20$$

Hence, the height at which the ball should be hit, is 20 m

OR

(b) In ΔDOC and ΔBOA, we have

$\angle ODC = \angle OBA$ (Alternate angles)

and $\angle OCD = \angle OAB$ (Alternate angles)

$\Rightarrow$ $\Delta DOC \sim \Delta BOA$ (By AA similarity)

$\Rightarrow$ $\frac{OD}{OB} = \frac{CD}{AB}$ (Corresponding parts of similar triangles)

$$\Rightarrow \frac{1}{2} = \frac{CD}{AB}\left(\because \frac{OD}{OB} = \frac{1}{2}\right) \Rightarrow AB = 2CD$$

Hence proved.

22. $\because$ Tangents drawn from an external point are equal in length.

$\therefore$ QC = CA, QD = BD and PA = PB

Since, QC = QD = 3 cm [Given]

$\Rightarrow$ CA = BD = 3 cm

Also, PC = PA – AC

$\Rightarrow$ PC = (12 – 3) cm = 9 cm [Given, PA = 12 cm]

Similarly, PD = 9D

$\therefore$ PC + PD = (9 + 9) cm = 18 cm

23. **(a)** We have, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ …(i)

Comparing equation (i) with $ax^2 + bx + c = 0$, we get

$a = \sqrt{3}$, $b = -2\sqrt{2}$ and $c = -2\sqrt{3}$

$\therefore\ b^2 - 4ac = (-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3}) = 8 + 24 = 32 > 0$

$$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow\ x = \frac{-(-2\sqrt{2}) \pm \sqrt{32}}{2\sqrt{3}} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow\ x = \frac{3\sqrt{2}}{\sqrt{3}}\sqrt{6} \text{ or } x = -\sqrt{\frac{2}{3}}$$

OR

(b) We have, $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

$\Rightarrow\ \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$

$\Rightarrow\ \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0$

$\Rightarrow\ (x + 3\sqrt{3})(\sqrt{3}x + 2) = 0$

$$\Rightarrow\ x = -3\sqrt{3} \text{ or } x = -\frac{2}{\sqrt{3}}$$

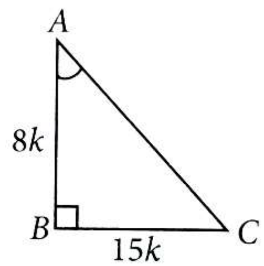
Hence, $-3\sqrt{3}$ and $-\frac{2}{\sqrt{3}}$ are the two roots of the given equation.

24. Consider a right triangle ABC, such that $\angle B = 90°$

Now, 15 cot A = 8 (Given)

$$\Rightarrow \cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let AB = 8k units and BC = 15k units



Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225K^2 = 289k^2 = (17k)^2$

$\Rightarrow\ AC = \sqrt{(17k)^2} = 17k$ units

$$\therefore\ \sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

$$\text{Thus, } \frac{24 \sec A}{25} = \frac{24}{25} \times \frac{17}{8} = \frac{51}{25}$$

25. We have numbers 1, 2, …., 99.

$\therefore$ Total number of possible outcomes = 99

Numbers divisible by 7 from 1 to 99 are 7, 14, …, 98

$\Rightarrow$ Number of favourable outcomes = 14

$$\therefore\ P(\text{Number divisible by 7}) = \frac{14}{99}$$

26.

English	Social Science	Hindi
192	240	168

We find HCF of 192, 240 and 168

$192 = 2^6 \times 3$

$240 = 2^4 \times 3 \times 5$

$168 = 2^3 \times 3 \times 7$

$\text{HCF} = 2^3 \times 3 = 24$

$\therefore$ Number of stacks for English books = $\frac{192}{24} = 8$

Number of stacks for social Science books = $\frac{240}{24} = 10$

Number of stacks for Hindi books = $\frac{168}{24} = 7$

27. α, β are zeroes of polynomial $3x^2 + 2x - 1$

Sum of zeroes = $\alpha + \beta = \frac{2}{3}$

Product of zeroes = $\frac{2}{3} = \frac{1}{3}$

(a) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$$= \frac{\left(-\frac{2}{3}\right)^3 - 3\left(-\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{-\frac{1}{3}} = \frac{-\frac{8}{27} - \frac{2}{3}}{-\frac{1}{3}} = \frac{-\frac{26}{27}}{\frac{-1}{3}} = \frac{26}{27} \times \frac{3}{1} = \frac{26}{9}$$

(b) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \left(-\frac{1}{3}\right)\left(-\frac{2}{3}\right) = \frac{2}{9}$

28. Given equations are: $2x + 3y = 7$

$(m - 1)x + (m + 1)y = (3m - 1)$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{m-1} = \frac{3}{m+1} = \frac{7}{3m-1}$

$\Rightarrow \frac{2}{m-1} = \frac{3}{m+1}$ and $\frac{3}{m+1} = \frac{7}{3m-1}$

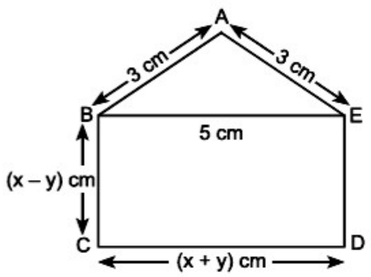
$\Rightarrow$ $2m + 2 = 3m - 3$ and $9m - 3 = 7m + 7$

$\Rightarrow$ $m = 5$ and m 5

So, $m = 5$

OR

Perimeter = 221 cm



$\Rightarrow$ AE + AB + BC + ED + DC = 21 cm

$\Rightarrow$ 3 + 3 + x – y + x – y + x + y = 21

$\Rightarrow$ 3x – y = 15 …(i)

and x + y = 5

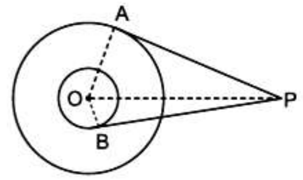
Solving (i) and (ii), we get x = 5, y = 0

29. Join OA, OB and OP. Now, AP = 24 cm, OA = 10 cm

$\angle OAP = 90^\circ$ [As, tangent is $\perp$ to the radius at the point of contact]

In ΔPAO, $OP^2 = (24)^2 + (10)^2$ [Pythagoras theorem]

$= 576 + 100 = 676$



Also PB is tangent to smaller circle, so $\angle OBP = 90^\circ$

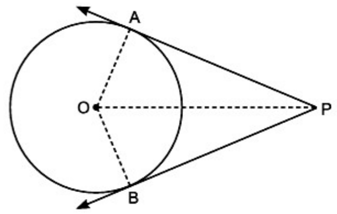
In ΔPBO, $OP^2 = OB^2 + PB^2$

$\Rightarrow 676 = (4)^2 + PB^2 \Rightarrow PB^2 = 676 - 16$

$\Rightarrow PB^2 = 660 \Rightarrow PB = 2\sqrt{165}$ cm

OR

Given: PA and PB are tangents drawn to a circle with centre O, from an external point P, with points of contact A and B respectively



So, $\angle OAP = \angle OBP = 90^\circ$

Now, in ΔOAP and ΔOBP

$\angle OAP = \angle OBP$

OP = OP

OA = OB

$\Rightarrow \Delta OAP \cong \Delta OBP$

$\Rightarrow$ PA = PB

30. As, $15\cot^2\theta + 4\operatorname{cosec}^2\theta = 23$

$\Rightarrow 15(\operatorname{cosec}^2\theta - 1) + 4\operatorname{cosec}^2\theta = 23$

$\Rightarrow 15\operatorname{cosec}^2\theta - 15 + 4\operatorname{cosec}^2\theta = 23$

$\Rightarrow 19\operatorname{cosec}^2\theta = 38$

$\Rightarrow \operatorname{cosec}^2\theta = 2$

$\Rightarrow 1 + \cot^2\theta = 2$

$\Rightarrow \cot^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{\cot^2\theta} = 1$

Now, $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

31.

% of female Teachers (C.I)	x_i	**No. of States/UT's (f_i)**	$u_i = \frac{x_i - A}{h} = \frac{x_i - 50}{10}$	f_iu_i
15 – 25	20	6	–3	–18
25 – 35	30	11	–2	–22
35 – 45	40	7	–1	–7
45 – 55	50 = A(say)	4	0	0
55 – 65	60	4	1	4
65 – 75	70	2	2	4
75 – 85	80	1	3	3
		$\sum f_i = 35$		$\sum f_iu_i = -36$

Here, A = 50 (say) and h = 10

$$\text{Mean} = A + \frac{\sum f_i u_i}{\sum f_i} \times h = 50 + \frac{(-36)}{35} \times 10 \qquad = 50 - \frac{72}{7} = \frac{278}{7} \approx 39.71$$

Mean percentage of female teachers is 39.71% (approx).

32. **(a)** For, 2x + 3y = 7

(p + q + 1) x + (p + 2q + 2) y = 4(p + q) + 1

We have, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

and $a_2 = p + q + 1$, $b_2 = p + 2q + 2$, $c^2 = -[4(p + q) + 1]$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{p+q+1} = \frac{3}{p+2q+2} = \frac{-7}{-[4(p+q)+1]}$$

Taking first two terms, we get

2p + 4q + 4 = 3p + 3q + 3

$\Rightarrow$ p – q – 1 = 0 …(i)

Taking last two terms, we get

12(p + q) + 3 = 7p + 14q + 14

$\Rightarrow$ 12P + 12q + 3 = 7p + 14q + 14

$\Rightarrow$ 5p – 2q –11 = 0 …(ii)

Solving (i) and (iii) for p and q using cross-multiplication method, we have

$$\frac{p}{11-2} = \frac{q}{-5+11} = \frac{1}{-2+5} \Rightarrow \frac{p}{9} = \frac{q}{6} = \frac{1}{3}$$

$$\Rightarrow \frac{p}{9} = \frac{1}{3} \text{ and } \frac{q}{6} = \frac{1}{3} \Rightarrow p = \frac{9}{3} = 3 \text{ and } q = \frac{6}{3} = 2$$

OR

(b) Let Vijay had x bananas in lot A and y bananas in lot B.

Case I: When x bananas are sold at the rate of Rs 2 for 3 bananas and y bananas are sold at the rate of Rs 1 per banana, then total cost = Rs 400

$$\Rightarrow \frac{2}{3}x + y = 400 \qquad \text{…(i)}$$

Case II: When x bananas are sold at the rate of Rs 1 per banana and y bananas at the rate of Rs 4 for 5 bananas, then total cost = Rs 460

$$\Rightarrow x + \frac{4}{5}y = 460 \qquad \text{….(ii)}$$

From (i), $y = 400 - \frac{2}{3}x$ ….(iii)

Using (iii) in (ii), we have

$$x + \frac{4}{5}\left(400 - \frac{2}{3}x\right) = 460$$

$$\Rightarrow x + 320 - \frac{8}{15}x = 460$$

$$\Rightarrow \frac{7x}{15} = 140 \Rightarrow x = \frac{15 \times 140}{7} = 300$$

Putting x = 300 in (iii), we get

$$y = 400 - \frac{2}{3}(300) = 400 - 200 = 200$$

$\therefore$ Total number of bananas = x + y = 500

33. Let the missing frequency be f and h = 3

Let us construct the following table for the given data

Class-interval	**Frequency (f_i)**	**Class mark (x_i)**	$u_i = \frac{x_i - 47.5}{h}$	$f_i u_i$
40 – 43	1	41.5	–2	–62
43 – 46	58	44.5	–1	–58
46 – 49	60	47.5 = a (let)	0	0
49 – 52	F	50.5	1	F
52 – 55	27	53.5	2	54
Total	$\sum f_i = 176 + f$			$\sum f_i u_i = f - 66$

$\therefore$ Mean = $a + h\left\{\frac{\sum f_i u_i}{\sum f_i}\right\}$

$\Rightarrow$ 47.2 = 47.5 + 3 × $\left\{\frac{f-66}{176+f}\right\}$ ($\because$ Given, mean = 47.2)

$\Rightarrow -0.3 = 3 \times \left\{\frac{f-66}{176+f}\right\}$

$\Rightarrow \frac{-1}{10} = \frac{f-66}{176+f}$

$\Rightarrow$ –176 – f = 10f – 660

$\Rightarrow$ 11f = 484 $\Rightarrow$ f = 4

Hence, the missing frequency is 44.

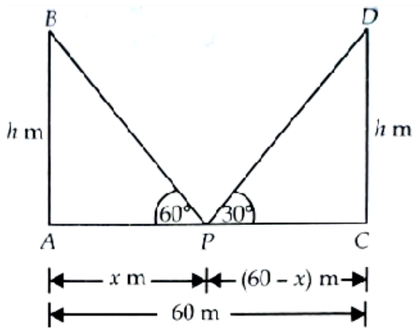
34. **(a)** Let AB and CD be two poles of equal height, h m.

Then AB = CD = h m

Let AP = x m

$\therefore$ CP = (60 – x) m

Now, in right ΔAPB,



$\frac{AB}{AP}$ = tan 60°

$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = x\sqrt{3}$

Again, in right ΔCPD, $\frac{CD}{CP}$ = tan 30°

$\Rightarrow \frac{h}{(60-x)} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{60-x}{\sqrt{3}}$

From (i) and (ii), we get $\sqrt{3}\ x = \frac{60 - x}{\sqrt{3}}$

$\Rightarrow$ 3x = 60 – x $\Rightarrow$ 4x = 60 $\Rightarrow$ x = 15

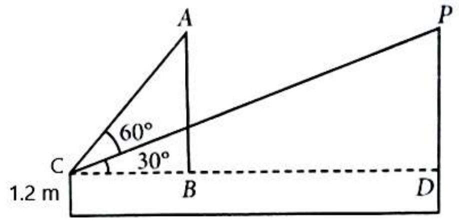
$\therefore$ CP = 60 – x = 60 – 15 = 45 m

Now, from (i), we have h = 15 × $\sqrt{3}$ = 15 × 1.732 = 25.98

Thus, the required point is 15 m away from the first pole and 45 m away from the second pole and height of each pole is 25.98 m

OR

(b) In the figure, let C be the position of the girl. A and P are two positions of the balloon. CD is the horizontal line from the eyes of the girl



Here, PD = AB = 88.2 m – 1.2 m = 87 m

In right ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}} m$$

In right ΔPDC,

$$\frac{PD}{CD} = \tan 30^\circ \Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3} m$$

Now, BD = CD – BC

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 87 \times \left(\frac{3-1}{\sqrt{3}}\right) = \frac{2 \times 87}{\sqrt{3}}$$

$$= \frac{2 \times 87 \times \sqrt{3}}{\sqrt{3}} = 2 \times 29 \times \sqrt{3} = 58\sqrt{3}\ m$$

Thus, the required distance between the two positions of the balloon is $58\sqrt{3}$ m .

35. The prime factorization of 60, 84 and 108 is

60 = 2 × 2 × 3 × 5, 84 = 2 × 2 × 3 × 7, 108 = 2 × 2 × 3 × 3 × 3

Number of students in each group subject to the given condition = HCF (60, 84, 108)

HCF (60, 84, 108) = 2 × 2 × 3 = 12

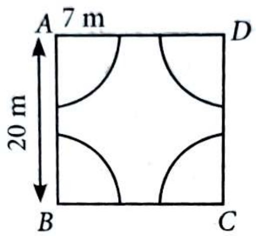
Number of groups in Music = $\frac{60}{12} = 5$

Number of groups in Dance = $\frac{84}{12} = 7$

Number of groups in Handicrafts = $\frac{108}{12} = 9$

Total number of rooms required = 5 + 7 + 9 = 21

36. (i) Area of square shaped grass field = 20 × 20 = 400 m^2



(ii) Area of field that horses can graze

$$= 4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7 = 154\ m^2$$

(iii) (a) Area of field grazed by one horse

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10$$

$$= \frac{1}{4} \times 314 = 78.5 \text{ m}^2$$

OR

(iii) (b) Area of the field that is left ungrazed

= Area of square shaped grass field – Area of field that horses can graze

= (400 × 100 × 100 – 154) cm^2

= 3999846 cm^2 = 399.9846 m^2 $\approx$ 400m^2

37. (i) Since, $\angle B = \angle D = 90°$, $\angle AMB = \angle CMD$

($\because$ Angle of incident = Angle of reflection)

$\therefore$ By AA similarity criterion, $\Delta ABM \sim \Delta CD$

(ii) Since, $\Delta ABM \sim \Delta CDM$

$\therefore$ $\angle A = \angle C = 30°$ [$\because$ Corresponding angles of similar triangles are also equal]

(iii) (a) Since, $\Delta ABM \sim \Delta CDM$

$$\therefore \quad \frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{1.8} = \frac{2.5}{1.5} \Rightarrow AB = \frac{2.5 \times 1.8}{1.5} = 3 \text{ m}$$

$\therefore$ Height of the building is 3 m

(iii) (b) Since, $\Delta ABM \sim \Delta CDM$

$$\therefore \quad \frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{6} = \frac{24}{8} \Rightarrow AB = 18 \text{ cm}$$

38. Let the numbers on the cards be a, a + d, a + 2d, …

According to question, we have

(a + 5d) + (a + 13d) = – 76

$\Rightarrow$ 2a + 18d = –76 $\Rightarrow$ a + 9d = – 38 …(i)

And (a + 7d) + (a + 15d) = –96

$\Rightarrow$ 2a + 22d = –96 $\Rightarrow$ a + 11d = – 48 ...(ii)

From (i) and (ii), we get 2d = – 10 $\Rightarrow$ d = –5

From (i), a + 9(– 5) = – 38 $\Rightarrow$ a = 7

(i) The difference between the numbers on any two consecutive cards = common difference of the A.P. = –5

(ii) Number on first card = a = 7

(iii) (a) Number on 19^{th} card = a + 18d = 7 + 18(–5) = –83

OR

(iii) (b) Number on 23^{rd} card = a + 22d = 7 + 22(–5) = –103

SOLUTIONS (Sample Paper – 3)

1. **(b)** All Natural numbers.
2. **(c)** 2, As the graph of y =p(x) intersects the x-axis at two distinct points.
3. **(b)** pq = 30
4. **(c)** Given,

$x^2 - 3x - (m + 2)(m + 5) = 0$

$\Rightarrow x^2 - (m + 5)x + (m + 2)x - (m + 2)(m + 5) = 0$

$\Rightarrow x[x - (m + 5)] + (m + 2)[x - (m + 5)] = 0$

$\Rightarrow [x - (m + 5)][x + (m + 2)] = 0$

$\Rightarrow x = (m + 5)$ or $x = -(m + 2)$

5. **(b)**
6. **(a)** $\frac{AB}{AD} = \frac{AC}{AE}$ [$\because$ By BPT]
7. **(c)** According to the question,

AB = AC

Using distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(x-0)^2 + (6-1)^2}$

Squaring both sides, we get

$(\sqrt{25+16})^2 = (\sqrt{x^2+25})^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\therefore \quad x = \pm 4$

8. **(a)** $\sqrt{x^2 + y^2}$
9. **(c)** $\tan\theta = \frac{\sqrt{3}-1}{1}$ $\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$

$\therefore P = \sqrt{3} - 1$

B = 1

By Pythagoras theorem;

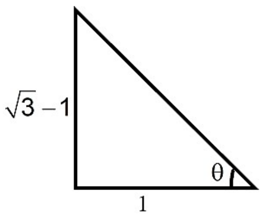
$H^2 = P^2 + B^2$

$H^2 = (\sqrt{3} - 1)^2 + (1)^2$

$H^2 = 3 + 1 - 2\sqrt{3} + 1$

$H^2 = 5 - 2\sqrt{3}$

$H = \sqrt{5 - 2\sqrt{3}}$



Now, $\sin\theta = \frac{P}{H}$

$\therefore \quad \sin\theta = \frac{\sqrt{3}-1}{\sqrt{5-2\sqrt{3}}}$

10. **(c)** As, $\theta = 30°$

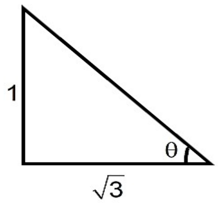
$\therefore 4\cos^3\theta - 3\cos\theta = 4\cos^3 30° - 3\cos 30°$

$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{3\sqrt{3}}{2} = 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 = \cos 90°$

11. **(b)** Given

$\cot\theta = \frac{B}{P}$



$\Rightarrow \cot\theta = \frac{\sqrt{3}}{1} = \cot 30°$

$\therefore \theta = 30°$

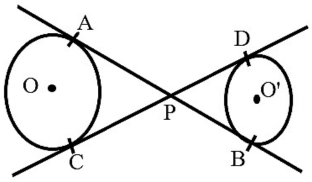
12. **(b)** The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

$\therefore \quad \angle POQ + \angle ROS = 180°$

$\therefore \quad \angle POQ + 110° = 180°$

$\therefore \quad \angle POQ = 70°$

13. **(b)** Given,



$\because$ The length of tangents drawn from an external point to a circle are equal.

$\therefore \quad PA = PC \quad \ldots(i)$

and $\quad PB = PD \quad \ldots(ii)$

On adding equations (i) and (ii), we get

PA + PB = PC + PD

AB = CD

14. **(b)** Area of sector = 231 cm^2

$\frac{\pi r^2\theta}{360°} = 231$

$\therefore \quad \frac{\pi r^2\theta}{180°} \times \frac{r}{2} = 231$

Length of arc $\times \frac{21}{2} = 231 \qquad \left[\because \text{Length of arc} = \frac{\pi r\theta}{180°}\right]$

Length of arc $= \frac{231 \times 2}{21}$

Length of arc = 22cm

15. **(d)** Perimeter of PQRS = Length of arc PS + length of arc QR + length of PQ + length of RS

$= \frac{22}{7} \times \frac{7 \times 40°}{180°} + \frac{22}{7} \times \frac{14 \times 40°}{180°} + (14 - 7) + (14 - 7)$

$= \frac{22 \times 40°}{180°}(1 + 2) + 14 = \frac{22 \times 2}{9} \times 3 + 14 = \frac{44}{3} + \frac{14}{1} = \frac{44 + 42}{3} = \frac{86}{3}\text{cm}$

16. **(b)** Lower limit = Class mark $-\frac{1}{2}\times$ Class size = $= 45 - \frac{1}{2} \times 10 = 45 - 5 = 40$

17. **(b)** $\frac{1}{2}$

18. **(d)** $\frac{4}{11}$

19. **(a)** Both assertion (A) and reason (R) are true and reason (R) is correct explanation of assertion(A).

20. **(b)** Both assertion (A) and reason (R) are true but reason (R) is not correct explanation of assertion (A).

21. $7 \times 11 \times 17 \times 19 + 19 = 19(7 \times 11 \times 17 + 1)$

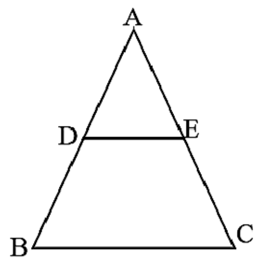
$= 19 \times 1310 = 2 \times 5 \times 19 \times 131$

Clearly there are more than 2 factors of the above number.

$\therefore$ $7 \times 11 \times 17 \times 19 + 19$ is a composite number.

22. In ΔABC, DE || BC (Given)

By BPT



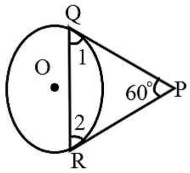
$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

$$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

23. Given,

We know that the lengths of tangents drawn from an external point to a circle are equal.



$\therefore$ PQ = PR

i.e. $\angle 1 = \angle 2$ [Angles opposite to equal sides of a Δ are equal]

In ΔPQR

$\angle P + \angle 1 + \angle 2 = 180^\circ$

$\Rightarrow 60^\circ + \angle 1 + \angle 1 = 180^\circ$

$\Rightarrow 2\angle 1 = 120^\circ$

$\Rightarrow \angle 1 = 60^\circ$

$\therefore \angle 1 = \angle 2 = 60^\circ$

i.e. All the three angles of this triangle are equal to 60°. Hence, ΔPQR is an equilateral triangle.

24. $$\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = \frac{2}{3}$$

$$\Rightarrow \frac{(\sec\theta + 1) - (\sec\theta - 1)}{(\sec\theta - 1)(\sec\theta + 1)} = \frac{2}{3}$$

$$\Rightarrow \frac{\sec\theta + 1 - \sec\theta + 1}{\sec^2\theta - 1^2} = \frac{2}{3}$$

$$\Rightarrow \frac{2}{\tan^2\theta} = \frac{2}{3}$$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60^\circ$ $(\because 0^\circ \le \theta \le 90^\circ)$

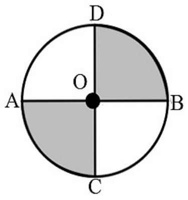
$\therefore$ $\theta = 60^\circ$

OR

$$\frac{(1 - \operatorname{cosec}^2\theta)(1 - \cos\theta)(1 + \cos\theta)}{1 - \sin^2\theta} = \frac{-(\operatorname{cosec}^2\theta - 1)(1^2 - \cos^2\theta)}{\cos^2\theta} = \frac{-\cot^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} = -1$$

$$= -\frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} = -1$$

25. Sector radius (r) = $\frac{14}{2} = 7\text{cm}$



$\theta = 90^\circ$ [As AB $\perp$ CD]

Total area of opposite sectors = 2 × area of one sector

$= 2 \times \frac{\pi r^2 \theta}{360^\circ}$ $= 2 \times \frac{22}{7} \times \frac{7 \times 7 \times 90^\circ}{360^\circ}$

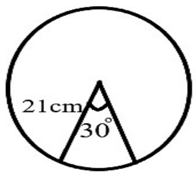
= 11 × 7 = 77cm^2

OR

Given, radius of circle (r) = 21cm

Central angle $\theta = 30^\circ$

$\therefore$ Length of arc $= \frac{\pi r \theta}{180^\circ}$



$= \frac{22}{7} \times \frac{21 \times 30^\circ}{180^\circ}$ = 11cm

26. Milk in 3 containers = 54 litres, 84 litres and 108 litres.

We have to find HCF of 54, 84 and 108 for measurement of larger cup that can measure the milk of above containers.

2	54
3	27
3	9
3	3
	1

2	84
2	42
3	21
7	7
	1

2	108
2	54
3	27
3	9
3	3
	1

54 = 2 × 3 × 3 × 3

84 = 2 × 2 × 3 × 7

108 = 2 × 2 × 3 × 3 × 3

HCF = 2 × 3 = 6 litres

$\therefore$ Measurement of required cup is 6 litres.

27. $3x^2 + 5x + 7$ having zeroes $\frac{m}{2}, \frac{n}{2}$ i.e. $\alpha = \frac{m}{2}$ and $\beta = \frac{n}{2}$

Sum of zeroes $(\alpha + \beta) = \frac{-b}{a}$

$\frac{m+n}{2} = \frac{-5}{3}$

$m + n = \frac{-10}{3}$ …(i)

and product of zeroes $(\alpha\beta) = \frac{c}{a}$

$\frac{m}{2} \times \frac{n}{2} = \frac{7}{3}$

$\frac{mn}{4} = \frac{7}{3}$

$mn = \frac{28}{3}$ …(ii)

Now, find the polynomial whose zeroes are (2m + 3n) and (3m + 2n)
Sum of zeroes = 2m + 3n + 3m + 2n = 5m + 5n

$= 5(m + n) = 5\left(\frac{-10}{3}\right)$ [from (i)]

Sum of zeroes $= \frac{-50}{3}$

Product of zeroes = (2m + 3n) × (3m + 2n) = $6m^2 + 4mn + 9mn + 6n^2$
$= 6m^2 + 6n^2 + 13mn = 6m^2 + 6n^2 + 12mm + mn$
$= 6(m^2 + n^2 + 2mn) + mn = 6(m + n)^2 + mn$

$= 6\left(\frac{-10}{3}\right)^2 + \frac{28}{3}$ [from (i) and (ii)]

$= 6 \times \frac{100}{9} + \frac{28}{3} = \frac{200}{3} + \frac{28}{3}$

Product of zeroes $= \frac{228}{3}$

∴ Required polynomial P(x) = k{x^2 – (sum of zeroes)x + product of zeroes}

$= k\left\{x^2 - \left(\frac{-50}{3}\right)x + \left(\frac{228}{3}\right)\right\}$

$= \frac{k}{3}(3x^2 + 50x + 228) = 3x^2 + 50x + 228$

28. (5k - 9)x + (2k - 3)y = 1 …(i)
(2k + 1)x + (4k - 3)y = 5 …(ii)
For infinitely many solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{5k-9}{2k+1} = \frac{2k-3}{4k-3} = \frac{1}{5}$

∴ $\frac{5k-9}{2k+1} = \frac{1}{5}$
⇒ 5(5k - 9) = 1(2k + 1)
⇒ 25k – 45 = 2k + 1
⇒25 – 2k = 1 + 45
⇒ 23k = 46
⇒ k = 2

$\frac{2k-3}{4k-3} = \frac{1}{5}$
⇒ 5(2k - 3) = 1(4k - 3)
⇒ 10k – 15 = 4k – 3
⇒ 10k – 4k = 15 – 3
⇒6k = 12
⇒ k = 2

∴ The value of k is 2.

OR

Let usual speed of the train b x km/h and time taken be y hours
Distance = speed × time
Distance = xy km
Case (I)
Speed of train = (x – 20)km/h
Time taken = (y + 2)hours
Distance = (x - 20)(y + 2)
= (xy – 20y + 2x - 40)km
∴xy – 20y + 2x – 40 = xy
⇒ 2x – 20y = 40
⇒x – 10y = 20 …(i)

Case (II)

Speed the train = (x + 10)km/h

Time taken = $\left(y-\frac{1}{2}\right)$hours

Distance = $(x+10)\times\left(y-\frac{1}{2}\right)=\left(xy-\frac{1}{2}x+10y-5\right)$km

$\therefore \quad xy = xy-\frac{1}{2}x+10y-5$

$\Rightarrow \quad -x + 20y = 10 \quad \ldots(ii)$

On adding equations (i) and (ii), we get

$$x-10y=20 \quad \ldots(i)$$
$$-x+20y=10 \quad \ldots(ii)$$
$$\overline{\quad 10y=30 \quad}$$

$\Rightarrow \quad y=\frac{30}{10}=3$

Putting the values of y in equation (i), we get

$x - 10 \times (3) = 20$

$\Rightarrow \quad x - 30 = 20$

$\Rightarrow \quad x = 50$

$\therefore$ Length of the journey = xy

= 50×3 = 150 km

29. $\angle C = 90°$

tan (C – B – A) = 0

$\Rightarrow$ tan (C – B – A) = tan 0

$\Rightarrow$ C – B – A = 0

$\Rightarrow$ $90° - B - A = 0 \quad \ldots(i)$

$\therefore$ A + B = 90°

and tan (B + C – A) = $\sqrt{3}$

$\therefore$ tan (B + C - A) = tan 60°

B + C – A = 60°

B + 90° - A = 60°

B – A = - 30° $\quad \ldots(ii)$

$\therefore$ A – B = 30°

On adding equations (i) and (ii), we get

$$A+B=90° \quad \ldots(i)$$
$$A-B=30° \quad \ldots(ii)$$
$$\overline{\quad 2A=120° \quad}$$

$\Rightarrow$ A = 60°

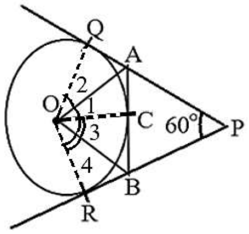
Putting the value of A in equation (i), we get

60° + B = 90°

$\Rightarrow$ B = 30°

$\therefore$ A = 60°, B = 30°

30. **Given :** A circle with centre O, PR and PQ are the tangents from an external point P, with point of contact R and Q respectively. Tangent AB with point of contact C where A and B are the points on PQ and PR respectively.



Construction : Join QO, RO, and CO.

Proof : In quadrilateral PQOR

$\angle PQO = \angle PRO = 90°$ (The tangent is $\perp$ to radius drawn through the point of contact)

$\therefore \angle P + \angle PQO + \angle PRO + \angle QOR = 360°$ (Angle sum property of a quadrilateral)

$\Rightarrow 60° + 90° + 90° + \angle QOR = 360°$

$\Rightarrow 240° + \angle QOR = 360°$

$\Rightarrow 240° + \angle QOR = 360°$

$\Rightarrow \angle QOR = 120°$

In ΔAQO and ΔACO

AQ = AC (Tangents drawn from an external point)

AO = AO (Common)

QO = CO (Radii)

$\therefore \Delta AQO \cong \Delta ACO$ (SSS congruent Rule)

$\angle 1 = \angle 2$ (CPCT)

Similarly $\angle 3 = \angle 4$

$\because \angle QOR = 120°$

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 = 120°$

$\Rightarrow \angle 1 + \angle 1 + \angle 3 + \angle 3 = 120°$

$\Rightarrow 2\angle 1 + 2\angle 3 = 120°$

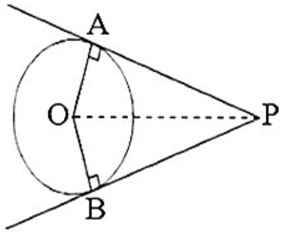
$\Rightarrow 2(\angle 1 + \angle 3) = 120°$

$\Rightarrow 2(\angle AOB) = 120°$

$\therefore \angle AOB = 60°$

OR

Given : A circle with centre O. PA and PB are two tangents drawn from an external point P.



To prove : Join AO, BO and PO.

Proof : In ΔPAO and ΔPBO

PO = PO [Common]

$\angle PAO = \angle PBO$ (Each 90° as tangent is $\perp$ to radius drawn through the point of contact)

AO = BO (Radii)

$\therefore \Delta PAO \cong \Delta PBO$ (RHS Congruent Rule)

Hence, PA $\cong$ PB (CPCT)

31.

Life time (in hours)	x_i	f_i	$d_i = x_i - A$	$f_i d_i$
1000 – 1100	1050	12	-500	-6000
1100 – 1200	1150	17	-400	-6800
1200 – 1300	1250	9	-300	-2700
1300 – 1400	1350	18	-200	-3600
1400 – 1500	1450	6	-100	-600
1500 – 1600	1550 = A	10	0	0
1600 – 1700	1650	7	100	700
1700 – 1800	1750	3	200	600
1800 – 1900	1850	16	300	4800
1900 – 2000	1950	2	400	800
		$\Sigma f_i = 100$		-12800

Let the assumed mean (A) be 1550

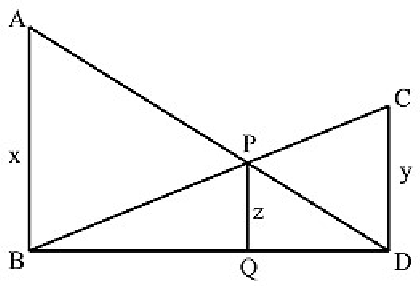
$\therefore$ Required men $\bar{x} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$$= 1550 + \left(\frac{-12800}{100}\right)$$

$$= 1550 - 128 = 1422 \text{ hours}$$

32. **Given :** In figure AB || CD || EF, AB = x units CD = y units, EF = z units

To prove :



Proof :

In ΔABD and ΔEFD

	AB || EF	(Given)
	$\angle ABD = \angle EFD$	(Corresponding angles)
	$\angle ADB = \angle EDF$	(Common)
$\therefore$	$\Delta ADB \sim \Delta EDF$	(AA similarity)
$\therefore$	$\dfrac{AB}{EF} = \dfrac{BD}{FD}$	(Corresponding sides of similar triangles)
$\therefore$	$\dfrac{BD}{FD} = \dfrac{x}{z} \Rightarrow \dfrac{FD}{BD} = \dfrac{z}{x}$	…(i)

Similarly $\Delta CDB \sim \Delta EFB$

$$\frac{CD}{EF} = \frac{BD}{BF}$$

$\therefore$ $\dfrac{BD}{BF} = \dfrac{y}{z} \Rightarrow \dfrac{BF}{BD} = \dfrac{z}{y}$ …(ii)

Now adding equation (i) and (ii), we get

$$\frac{FD}{BD} + \frac{BF}{BD} = \frac{z}{x} + \frac{z}{y}$$

$$\frac{FD+BF}{BD}=z\left[\frac{1}{x}+\frac{1}{y}\right]$$

$$\frac{BD}{BD}=z\left[\frac{1}{x}+\frac{1}{y}\right]$$

$\therefore \quad \frac{1}{z}=\frac{1}{x}+\frac{1}{y}$ Hence proved

33. $25x^2 - 15(a-b)x + 2(a-b)^2 - ab = 0$

$\Rightarrow 25x^2 - 15(a-b)x + 2(a^2 - b^2 - 2ab) - ab = 0$

$\Rightarrow 25x^2 - 15(a-b)x + 2a^2 + 2b^2 - 4ab - ab = 0$

$\Rightarrow 25x^2 - 15(a-b)x + 2a^2 - 4ab + 2b^2 - ab = 0$

$\Rightarrow 25x^2 - 15(a-b)x + 2a(a-2b) - b(a-2b) = 0$

$\Rightarrow 25x^2 - 15(a-b)x + (a-2b)(2a-b) = 0$

$\Rightarrow 25x^2 - 5(a-2b)x - 5(2a-b) + (a-2b)(2a-b) = 0$

$\Rightarrow 5x[5x-(a-2b)] - (2a-b)[5x-(a-2b)] = 0$

$\Rightarrow [5x-(a-2b)][5x-(2a-b)] = 0$

$\Rightarrow 5x - (a-2b) = 0$ **Or** $5x - (2a-b) = 0$

$\Rightarrow 5x = a - 2b$ **or** $\Rightarrow 5x = 2a - b$

$\Rightarrow x = \frac{a-2b}{5}$ **or** $\Rightarrow x = \frac{2a-b}{5}$

OR

$$\frac{1}{2a+b+x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{x}$$

$\Rightarrow \quad \frac{1}{2a+b+x}-\frac{1}{x}=\frac{1}{2a}+\frac{1}{b}$

$\Rightarrow \quad \frac{x-2a-b-x}{x(2a+b+x)}=\frac{b+2a}{2ab}$

$\Rightarrow \quad \frac{-(2a+b)}{x(2a+b+x)}=\frac{(2a+b)}{2ab}$

$\Rightarrow \quad x(2a + b + x) = -2ab$

$\Rightarrow \quad 2ax + bx + x^2 = -2ab$

$\Rightarrow \quad x^2 + 2ax + bx + 2ab = 0 \Rightarrow x(x + 2a) + b(x + 2a) = 0$

$\Rightarrow \quad (x + 2a)(x + b) = 0$

$\therefore \quad x = -2a$ or $x = -b$

34.

Monthly Income (in ₹)	**Monthly Income (in ₹) Class – interval**	**Number of Workers**	**f**	**cf**
Income more than ₹ 12000	12000 – 15000	60	13	13
Income more than ₹ 15000	15000 – 18000	47	8	21
Income more than ₹ 18000	18000 – 21000	39	18	39
Income more than ₹ 21000	21000 – 24000	21	17	56
Income more than ₹ 24000	24000 – 27000	4	3	59
Income more than ₹ 27000	27000 – 30000	1	1	60
Income more than ₹ 30000	30000 – 33000	0	0	60

Now, $\frac{n}{2}=\frac{60}{2}=30$

$\therefore$ 1800 – 2100 is the median class.

$f = 18, h = 3000$

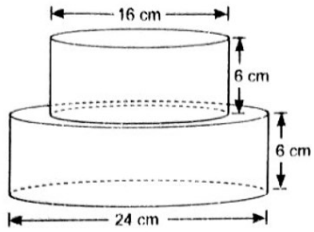
cf = 21

l = 18000

$\therefore$ Median monthly income $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h = 18000 + \left(\dfrac{30-21}{18}\right) \times 3000$

$$= 18000 + \frac{9}{18} \times 3000 = 18000 + 1500 = \text{Rs}\,19500$$

35. Radius of large cylindrical cake (R) = 12 cm



Radius of small cylindrical cake (r)= 8 cm

Height of each cylindrical cake (h) = 6cm

Area of cake covered with cream =CSA of larger cylindrical cake + CSA of smaller cylindrical cake + area of top of smaller cylinder + area of top ring of larger cylinder

$= 2\pi Rh + 2\pi rh + \pi r^2 + \pi (R^2 - r^2)$

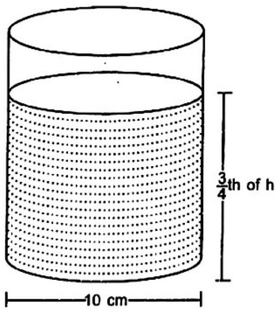
$= 2\pi h (R+r) + \pi r^2 + \pi R^2$

$= 2\pi h (R+r) + \pi R^2$

$= 2 \times \frac{22}{7} \times 6 \times (12+8) + \frac{22}{7} \times 12 \times 12$

$= \frac{22}{7}(12 \times 20 + 144) = \frac{22}{7} \times 384 = 1206.86 \text{cm}^2$

OR



Radius of cylindrical vessel $(R) = \frac{10}{2} = 5\text{cm}$

Let the height of cylindrical vessel be h cm

Radius of spherical stone $(r) = \frac{2}{2} = 1\text{cm}$

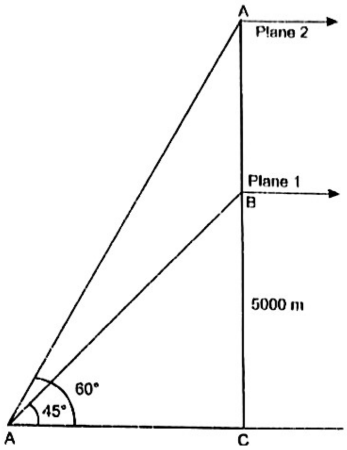
$\therefore$ ATQ, $\frac{1}{4}$(volume of cylinder) = 50 $\times$ volume of each spherical stone

$\frac{1}{4}\pi R^2 h = 50 \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad \frac{1}{4} \times 5 \times 5 \times h = 50 \times \frac{4}{3} \times (1)^3$

$\Rightarrow \quad h = \frac{32}{3}\text{cm}$

36.



There are two planes, plane 1 and plane 2 and plane 2 is flying above plane 1 at 20 m/s.

(i) Speed of plane 1 = 10 m/s

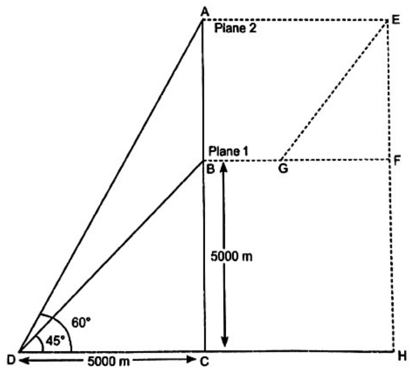
time 20 sec.

Distance covered by plane 1 = speed $\times$ time = $10 \times 20 = 200$ m

Speed of plane 2 = 20m/s

time 20 sec.

Distance covered by plane 2 = $20 \times 20 = 400$ m



(ii) AE= 400 m

BG= 200 m

GF = AE – BG [$\because$ AE = BF]

= 400m – 200m = 200 m

InΔACD, $\tan 60° = \frac{AC}{DC}$

$\Rightarrow \quad \sqrt{3} = \frac{AB + 5000}{5000}$

$\Rightarrow \quad AB = 5000(\sqrt{3} - 1) = 3650$

AB = EF = 3650m

$\therefore$ GE = $EF^2 + GF^2$

= $(3650)^2 + (200)^2$ = 13322500 + 40000

$\Rightarrow$ GE^2 = 13362500

$\therefore$ GE = 3655.47 m

Distance between both planes = 3655.47 m

(iii) In ΔBCD, $\frac{BC}{DC} = \tan 45°$

$\Rightarrow \quad \frac{5000}{DC} = 1$

$\therefore$ DC = 5000 m

In ΔACD, $\frac{AC}{DC} = \tan 60°$

$\Rightarrow \quad \frac{AC}{5000} = \sqrt{3}$

$\Rightarrow \quad AC = 5000\sqrt{3}m = 8650m$

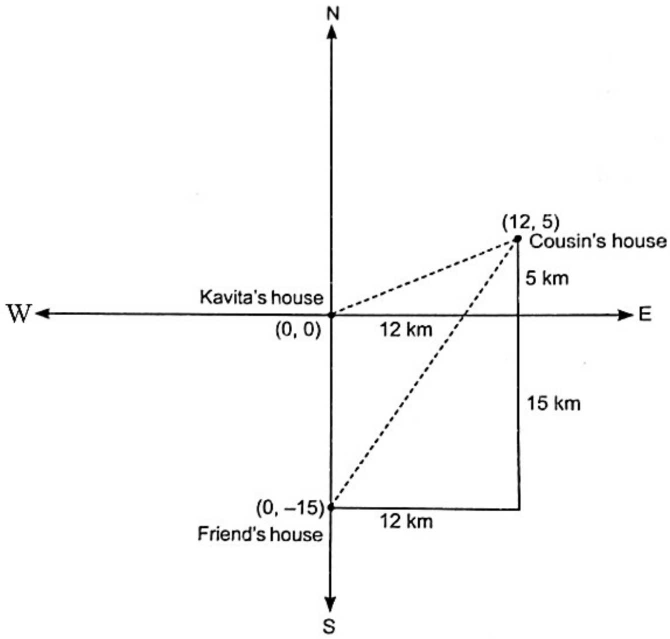
$\therefore$Height of above plane from ground = 8650 m

OR

Distance between both planes, AB = AC – BC

= 8650 m – 5000 m = 3650 m

37.



(i) Coordinates of cousin's house = (12, 5)

Coordinates of friend's house = (0, -15)

(ii) Distance between Kavita's house and friend's house = 15 km

(iii) Distance between Kavita's house and cousin's house

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad [\because \text{ Distance formula}]$$

$$= \sqrt{(12-0)^2 + (5-0)^2}$$

$$= \sqrt{144+25} = \sqrt{169} = 13\text{km}$$

OR

Distance between cousin's house and friend's house

$$= \sqrt{(12-0)^2 + [5-(-15)]^2}$$

$$= \sqrt{12^2 + 20^2} = \sqrt{144+400}$$

$$= \sqrt{544} = 23.32\text{km}$$

38. (i) From the given sequence of pearls and glasses, Arithmetic Progression series isobtained.

(ii) Number of glass in 1st string (a_1) = 1

Number of glass in 2nd string (a_2) = 2

$d = a_2 - a_1$

$= 2 - 1 = 1$

Number of glasses used in 9th string from last = Number of glasses used in 9th string from first

$= a + 8d$

$= 1 + 8 \times$

$1 = 1 + 8 = 9$

(iii)Total glasses used in all the strings = Number of glasses used in 1st 9 strings

+ Number of glasses used in last 9 strings

+ Number of glasses used in 10th string

$$= 2 \times \frac{n}{2}[2a + (n-1)d] + 10$$

[$\because$ Number of glasses in 1st 9 strings = Number of glasses in last 9 strings]

$$= 2 \times \frac{9}{2}[2 \times 1 + (9 - 1)1] + 10$$

= 9[2 + 8] + 10 = 90 + 10 = 100 glasses

OR

Total number of pearls in strings = number of pearls used in 1st 9 strings
+ number of pearls used in last 9 strings
+ number of pearls in 10th string

Number of pearls in 1st string = 100

Number of pearls in 2nd string = 98

$\therefore$ d = 2 till 9th string

[$\because$ Number of pearls in 1st 9 strings = Number of pearls in last 9 strings]

Number of pearl in 10th string (a) = a + 9d

= 100+ 9(-2)

= 100 – 18= 82

$\therefore$ Total number of pearls used in all strings = $= 2\left[\frac{n}{2}(2a + (n - 1)d\right]$ + Number of pearls in 10th string

$$= 2\left[\frac{9}{2}(2 \times 100 + (9 - 1)(-2)\right] + 82$$

= 9(200 – 16) + 82

= 1656 +82 = 1738

SOLUTIONS (Sample Paper – 4)

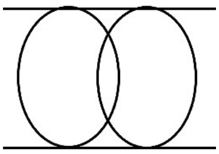
1. **(c)** Common difference (d) of A.P. $\sqrt{18}, \sqrt{50}, \sqrt{95}$,…., is given by

$\sqrt{50}-\sqrt{18}=\sqrt{2\times 25}-\sqrt{2\times 9}=5\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

$\therefore$ Fourth term, a_4 = a + 3d = $\sqrt{18}+3\times(2\sqrt{2})$

$=3\sqrt{2}+6\sqrt{2}=9\sqrt{2}=\sqrt{2\times 81}=\sqrt{162}$

2. **(c)**



Hence, two common tangents can be drawn.

3. **(d)** Here, r = 18cm, $\theta = 60^\circ$

Length of arc $=\frac{\theta}{360^\circ}\times 2\pi r=\frac{60^\circ}{360^\circ}\times 2\pi\times 18=6\pi\text{cm}$

4. **(a)** Let the other number be x.

Product of two numbers = HCF $\times$ LCM of two numbers

$\therefore x\times 161 = 23\times 1449$

$\Rightarrow x=\frac{23\times 1449}{161}\Rightarrow x=207$

5. **(d)** The distance between points P(a cosθ + b sinθ, 0) and Q(0,a sin θ - b cosθ) is given by

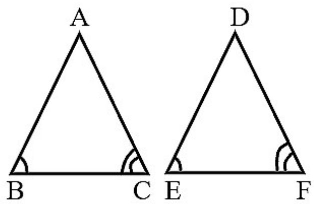
$PQ=\sqrt{(a\cos\theta+b\sin\theta-0^2)+(0-a\sin\theta+b\cos\theta)^2}$

$=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta^2+2ab\cos\theta\sin\theta+a^2\sin^2\theta+b^2\cos^2\theta-2ab\cos\theta\sin\theta}$

$=\sqrt{a^2(\cos^2\theta+\sin^2\theta)+b^2(\sin^2\theta+\cos^2\theta)}=\sqrt{a^2+b^2}$

6. **(b)** In ΔABC and ΔDEF, $\angle$B = $\angle$E, $\angle$F = $\angle$C and AB = 3DE

We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.



Since, AB $\neq$ DE

Therefore, ΔABC and ΔDEF are not congruent.

7. **(c)** Value of sinθ is always less than or equal to 1.

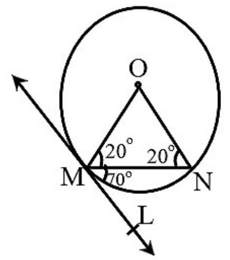
8. **(a)** The radius of the greatest sphere that can be cut off from the cylinder = 1 cm

$\therefore$ Volume of the sphere $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi(1)^3=\frac{4}{3}\pi\text{cm}^3$

9. **(c)** The number of zeroes of the polynomial f(x) lying between – 4 and 4 is 4. As between – 4 and 4, the graph intersects the x-axis at four points.

10. **(d)** For probability of an event E, $0\leq P(E)\leq 1$.

11. **(b)**



$\angle OML = 90°$ (Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$\angle OMN = 90° - 70° = 20°$

Now, In ΔOMN,

$OM = ON$ (radii of circle)

$\Rightarrow \angle OMN = \angle ONM = 20°$ (angles opposts to equal sides are equal)

In ΔOMN, $\angle OMN + \angle ONM + \angle MON = 180°$

$\therefore \angle MON = 180° - (20° + 20°) = 140°$

12. **(b)** $C.S.A. = 2\pi rh = 2 \times \frac{22}{7} \times r \times 14 = 264$ (given)

$\Rightarrow \quad r = \frac{264}{88} = 3cm$

$\therefore$ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 3 \times 3 \times 14$

$= 396\ cm^3$

13. **(c)** Here, $d_i = x_i - 13 \Rightarrow a = 13$ [$\because\ d_i = x_i - a$]

Also, $\Sigma f_i d_i = 30$ and $\Sigma f_i = 120$

Now, mean, $\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 13 + \frac{30}{120} = 13.25$

14. **(b)** Since the given system of equations have no solution.

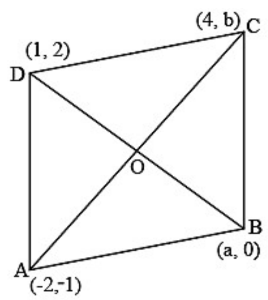
$\therefore \quad \frac{3}{6} = \frac{1}{k} \neq \frac{3}{8}$

Now, $\frac{3}{6} = \frac{1}{k} \Rightarrow k = 2$

Clearly, $\frac{1}{k} \neq \frac{3}{8}$ for $k = 2$

15. **(a)** The given points are A(-2, -1), B(a, 0), C(4, b) and D(1, 2). Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of P are



$X = \frac{-2+4}{2} = \frac{1+a}{2} \Rightarrow a = 1$

and $Y = \frac{-1+b}{2} = \frac{0+2}{2} \Rightarrow b = 3$

16. **(a)** In triangles ABC and DEF, we have

$$\frac{AB}{DF}=\frac{3.8}{7.6}=\frac{1}{2},\frac{BC}{FE}=\frac{6}{12}=\frac{1}{2},\frac{CA}{ED}=\frac{3\sqrt{3}}{6\sqrt{3}}=\frac{1}{2}$$

$$\therefore \frac{AB}{DF}=\frac{BC}{FE}=\frac{CA}{ED}=\frac{1}{2}$$

$\Rightarrow \Delta ABC \sim \Delta DFE$ (By SSS similarity criterion)

$\Rightarrow \angle A = \angle D$, $\angle B = \angle F$ and $\angle C = \angle E$ (Corresponding angles of similar triangles)

Hence, $\angle F = 60°$

17. **(a)** Given, $2\sin 2\theta = 1 \Rightarrow \sin 2\theta = ½$

$\Rightarrow 2\theta = 30°$ $\left(\because \sin 30° = \frac{1}{2}\right)$

$\Rightarrow \theta = 15°$

18. **(a)** Total number of possible outcome = 52

$\because$ Number of red cards = 26

So, number of favourable outcomes = 26

$\therefore$ Required probability $=\frac{26}{52}=\frac{1}{2}$

19. **(d)** Let P(6, -6) be the given point and O(0, 0) be the origin.

Then, $OP=\sqrt{(6-0)^2+(-6-0)^2}=\sqrt{6^2+(-6)^2}=6\sqrt{2}$

So, Assertion (A) is false but Reason (R) is true.

20. **(c)** Favourable outcomes always lies in the sample space of total number of outcomes. So, Reason (R) is false. Total possible outcomes are {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT} i.e., 8

Let E be the event of getting exactly 1 head.

$\therefore$ Outcomes favourable to E are {TTH, THT, HTT} i.e., 3

$\therefore P(E)=\frac{3}{8}$

$\therefore$ Assertion (A) is true.

21. In ΔABC, DE || BC (Given)

$\therefore \frac{AD}{DB}=\frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{x}{x+1}=\frac{x+3}{x+5} \Rightarrow x(x+5)=(x+3)(x+1)$

$\Rightarrow x^2 + 5x = x^2 + 4x + 3 \Rightarrow x = 3$

22. (a) We have, $5(x - 3)^2 = 20$

$\Rightarrow (x - 3)^2 = 4 \Rightarrow x^2 - 6x + 9 = 4$

$\Rightarrow x^2 - 6x + 5 = 0 \Rightarrow x^2 - 5x - x + 5 = 0$

$\Rightarrow x(x - 5) - 1(x - 5) = 0 \Rightarrow (x - 5)(x - 1) = 0$

$\Rightarrow x - 5 = 0$ or $x - 1 = 0 \Rightarrow x = 5$ or $x = 1$

OR

(b) Given equation is, $px^2 - 14x + 8 = 0$.

Let roots of equation be α and β such that

$\beta = 6\alpha \Rightarrow 6\alpha - \beta = 0$ …(i)

Now, sum of roots $=\alpha+\beta=-\left(\frac{-14}{p}\right)=\frac{14}{p}$ …(ii)

and product of roots $=\alpha\beta=\frac{8}{p}$ …(iii)

Solving (i) and (ii), we get $\alpha = \frac{2}{p}$ and $\beta = \frac{12}{p}$

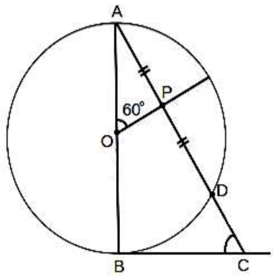
Putting these values in (iii) we get

$\left(\frac{2}{p}\right) \times \left(\frac{12}{p}\right) = \frac{8}{p} \Rightarrow 8p = 24 \Rightarrow p = 3 \quad (\because p \neq 0)$

23. Given, OP bisects the chord AD

$\therefore$ OP $\perp$ AD [$\because$ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

$\Rightarrow$ m$\angle$P = 90°



Since, BC is the tangent to the circle at point B.
Therefore, OB is perpendicular to BC.

$\therefore$ m$\angle$B = 90°

Now, m$\angle$BOP + m$\angle$AOP = 180° [Linear pair]

$\Rightarrow$ m$\angle$BOP + 60° = 180° [$\because$ Given, m$\angle$AOP = 60°]

$\Rightarrow$ m$\angle$BOP = 120°

In a quadrilateral BOPC,

m$\angle$B + m$\angle$BOP + m$\angle$P + m$\angle$C = 360° [$\because$ Sum of the angles of a quadrilateral is 360°]

$\Rightarrow$ 90° + 120° + 90° + m$\angle$C = 360°

$\Rightarrow$ m$\angle$C = 360° - 300° = 60°

Hence, m$\angle$C = 60°

24. (a) Put $\theta = 45°$ in $2\sec^2\theta + 3\ \text{cosec}^2\theta - 2\sin\theta\cos\theta$

$= 2\sec^2(45°) + 3\text{cosec}^2(45°) - 2\sin(45°)\cos(45°)$

$= 2(\sqrt{2})^2 + 3(\sqrt{2})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 4 + 6 - 1 = 9$

OR

(b) We have, $\frac{2\tan 30°.\sec 60°.\tan 45°}{1-\sin^2 60°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right).(2)(1)}{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{4}{\sqrt{3}}}{1-\frac{3}{4}} = \frac{\frac{4}{\sqrt{3}}}{\frac{1}{4}} = \frac{4}{\sqrt{3}} \times 4 = \frac{16}{\sqrt{3}}$$

25. For Jyanti, total number of possible outcomes = 36
Favourable outcome is (6, 6) i.e., 1.

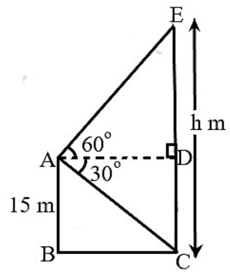
$\therefore$P(getting the number 36) $= \frac{1}{36}$

For Pihu, total number of possible outcomes = 6
Favourable outcome is 6 i.e., 1

$\therefore$P(getting the number 36) $= \frac{1}{6}$

$\therefore$Pihu has the better chance.

26. Let AB be the building of height 15m and CE be the cable tower of height h m.
Now, CD = AB = 15m, DE = CE – CD = (h – 15)m



In right ΔADE,

$$\tan 60^\circ = \frac{DE}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{h-15}{AD} \Rightarrow AD = \frac{h-15}{\sqrt{3}} \quad \ldots(i)$$

In right ΔACD,

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{AD} \Rightarrow AD = 15\sqrt{3} \quad \ldots(ii)$$

From (i) and (ii), we get $\frac{h-15}{\sqrt{3}} = 15\sqrt{3}$

$\Rightarrow$ h – 15 = 45 $\Rightarrow$ h = 60

Thus, the height of the tower is 60m.

27. (a) Given linear equations are

$$3x + 5y = 15 \Rightarrow y = \frac{15-3x}{5} \quad \ldots(i)$$

$$\text{and } 6x - 5y = 30 \Rightarrow y = \frac{6x-30}{5} \quad \ldots(ii)$$

Let us find at least two solutions for each of the equations (i) and (ii)

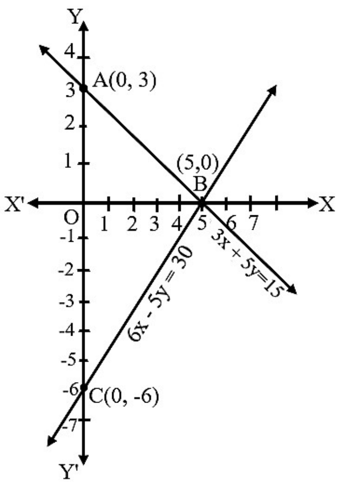
x	0	5
y	3	0

(i)

x	0	5
y	-6	0

(ii)

Now, let us plot the points A(0, 3), B(5, 0) on the graph paper and join them to get line 3x + 5y = 15.
Similarly, plot the points C(0, -6), B(5, 0) on the graph paper and join them to get the line 6x – 5y = 30.
The graphical representation of the given pair of linear equations is given below:



Clearly, the lines intersect each other at the point (5, 0).

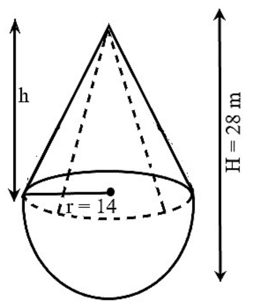
OR

(b) Let unit's digit be y and ten's digit be x. Then, number formed = 10x + y.
Number obtained by reversing the digits = 10y + x

Now, according to the question, $x + y = 8$ …(i)
and $(10x + y) – (10y + x) = 18 \Rightarrow 9x – 9y = 18$
$\Rightarrow x – y = 2$ …(ii)
From (i), $y = 8 – x$ …(iii)
Substituting the value of y from (iii) in (ii), we get
$x – (8 – x) = 2 \Rightarrow 2x – 8 = 2 \Rightarrow 2x = 10 \Rightarrow x = 5$
Substituting $x = 5$ in (iii), we get
$y = 8 – 5 = 3$
$\therefore x = 5, y = 3$
$\therefore$ Required number $= 10x + y = 10(5) + 3 = 50 + 3 = 53$.

28. Let the sides of the two squares be x cm and y cm, where $x > y$.
Then their area are x^2 and y^2 and their perimeters are 4x and 4y.
By the given condition, $x^2 + y^2 = 544$ and $4x – 4y = 32$
$\Rightarrow x – y = 8$
$\Rightarrow x = y + 8$ …(i)
Substituting the value of x in $x^2 + y^2 = 544$, we get
$(y + 8)^2 + y^2 = 544$
$\Rightarrow y^2 + 64 + 16y + y^2 = 544$
$\Rightarrow 2y^2 + 16y – 480 = 0 \Rightarrow y^2 + 8y – 240 = 0$
$\Rightarrow y^2 + 20y – 12y – 240 = 0$
$\Rightarrow y(y + 20) – 12(y + 20) = 0 \Rightarrow (y – 12)(y + 20) = 0$
$\Rightarrow y = 12$ ($\because y \neq -20$ as length cannot be negative)
From (i), $x = 12 + 8 = 20$
Thus, the sides of the two squares are 20cm and 12cm.

29. Total height of vessel (H) = 28m
Radius of hemisphere (r) = 14m
$\therefore$ Height of conical tent (h) = H – r
= 28 – 14 = 14m
Here, radius of base of cone (r) = radius of the hemisphere (r) = 14 m



Slant height, $l = \sqrt{r^2 + h^2}$
$= \sqrt{(14)^2 + (14)^2} = 14\sqrt{2}$m
$\therefore$ Area of metal sheet required = Curved surface area of hemisphere $= 2\pi r^2$
$= 2 \times \frac{22}{7} \times 14 \times 14 = 1232 m^2$
Also, area of canvas required = Curved surface area of cone
$\pi rl = \frac{22}{7} \times 14 \times 14\sqrt{2} = 44 \times 14\sqrt{2} = 616\sqrt{2} m^2$

30. (a) Let A(4, 2), B(7, 5) and C(9, 7) be the given points
Using distance formula, we have
$AB = \sqrt{(7-4)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$units

$BC = \sqrt{(9-7)^2 + (7-5)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$units

and $CA = \sqrt{(4-9)^2 + (2-7)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$ units

Now, AB + BC = $= 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$ units

i.e., AB + BC = CA

$\therefore$ The given point are collinear.

Hence, the given points does not form a triangle.

OR

(b) Let the coordinates of point P be (2y, y).

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points then distance between A and B is :

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Since P(2y, y) is equidistant from Q(2, -5) and R(-3, 6).

$\therefore$ PQ = PR $\Rightarrow PQ^2 = PR^2$

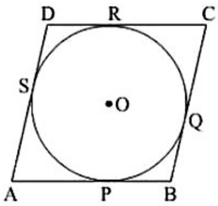
$\Rightarrow (2y - 2)^2 + (y + 5)^2 = (2y + 3)^3 + (y - 6)^2$

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow 29 + 2y = 45 \Rightarrow 2y = 16 \Rightarrow y = 8$

Hence, coordinates of point P are (16, 8).

31. Given, a parallelogram ABCD circumscribes a circle with centre O.



To prove that ABCD is a rhombus i.e., AB = BC = CD = AD.

Now, AP = AS (Tangents drawn from A) …(i)

BP = BQ (Tangents drawn from B) …(ii)

CR = CQ (Tangent drawn from C) …(iii)

DR = CQ (Tangents drawn from D) …(iv)

On adding (i), (ii), (iii) and (iv) we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ 2AB = 2AD (Since, ABCD is a parallelogram $\therefore$ AB = CD and AD = BC)

$\Rightarrow$ AB = AD

$\therefore$ AD = BC = CD = AB

$\Rightarrow$ ABCD is a rhombus. Hence proved.

32. Let sister's present age be x years then girls's present age be 2x years.

According to given condition.

$(x + 4)(2x + 4) = 160$

$\Rightarrow \quad 2x^2 + 4x + 8x + 16 = 160$

$\Rightarrow \quad 2x^2 + 12x - 144 = 0$

$\Rightarrow \quad x^2 + 6x - 72 = 0$

$\Rightarrow \quad (x + 12)(x - 6) = 0$

$\Rightarrow \quad x + 12 = 0$ or $x - 6 = 0$

$\Rightarrow \quad x = -12$ (rejected) or $x = 6$

$\therefore$ Sister's present age = 6 years

Girls's present age = 12 years

OR

Since, $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, then it must satisfy it.

So, $3(-2)^2 + 7(-2) + p = 0$

$\Rightarrow 12 - 14 + p = 0$

$\Rightarrow p = 2$

Given equation is,

$x^2 + k(4x + k - 1) + p = 0$

$\Rightarrow \quad x^2 + 4kx + k(k - 1) + 2 =$ $\quad [\because p = 2]$

As roots are equal, then D = 0

$\Rightarrow (4k)^2 - 4 \times 1 \times [k(k - 1) + 2)] = 0$

$\Rightarrow 16k^2 - 4(k^2 - k + 2) = 0$

$\Rightarrow 16k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow 12k^2 + 4k - 8 = 0$

$\Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$

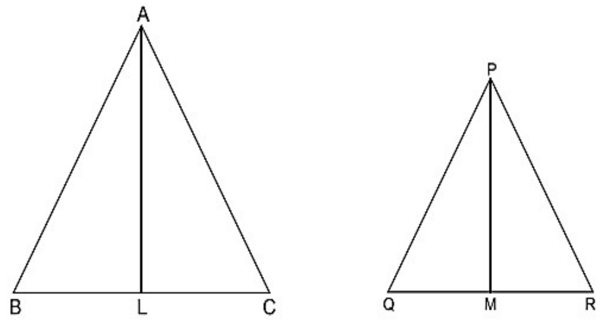
$\Rightarrow 3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow (3k - 2)(k + 1) = 0$

$\Rightarrow 3k - 2 = 0$ or $k + 1 = 0$

$\Rightarrow k = \frac{2}{3}$ or -1

33. **Given :**ΔABC and ΔPQR such that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AL}{PM}$ where AL and PM are the medians of ΔABC and ΔPQR respectively.



To prove :$\Delta ABC \sim \Delta PQR$

Proof : We have, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AL}{PM}$

$\frac{AB}{PQ} = \frac{2BL}{2QM} = \frac{AL}{PM}$ $\quad (\because$ AL and PM are medians)

$\frac{AB}{PQ} = \frac{BL}{QM} = \frac{AL}{PM}$ $\quad$ …(i)

In ΔABL and ΔPQM,

$\frac{AB}{PQ} = \frac{BL}{QM} = \frac{AL}{PM}$ $\quad$ [using (i)]

$\therefore \quad \Delta ABL \sim \Delta PQM$ $\quad$ (SSS similarity)

$\Rightarrow \quad \angle B = \angle Q$ $\quad$ [Corresponding angles of similar Δ's are equal] …(ii)

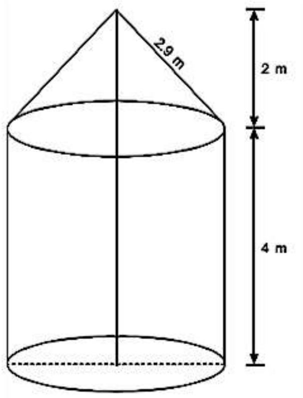
In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{BC}{QR}$ $\quad$ (Given)

$\Rightarrow \quad \angle B = \angle Q$ $\quad$ ...[From (ii)]

$\Rightarrow \quad \Delta ABC \sim \Delta PQR$ $\quad$ …(SAS similarity)

34. Let 'r' be the common base radius each of the conical and the cylindrical portion.
Height of conical portion, h_1 = 2m



Slant height of conical portion, l = 2.9m

Now, $l = \sqrt{r^2 + h_1^2}$

$\Rightarrow \quad 2.9 = \sqrt{r^2 + (2)^2}$

$\Rightarrow \quad 8.41 = r^2 + 4$

$\Rightarrow \quad r^2 = 4.41$

$\Rightarrow \quad r = 2.1\text{m}$

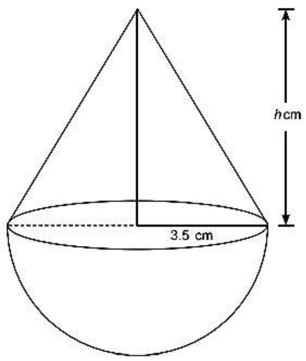
Now, height of cylindrical portion, h = 4m

Area of canvas needed = $2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times 2.1 \times 4 + \frac{22}{7} \times 2.1 \times 2.9$

= 52.8 + 19.14 = 71.94 m^2

OR



For hemisphere : Radius, r = 3.5 cm
For cone : Radius of base, r = 3.5 cm
Let height be h cm
Total volume = Volume of hemisphere + Volume of cone

$\Rightarrow \quad 166\frac{5}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad 166\frac{5}{6} = \frac{2}{3}\pi(3.5)^3 + \frac{1}{3}\pi(3.5)^2 h$

$\Rightarrow \quad \frac{1001}{6} = \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 + \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times h$

$\Rightarrow \quad 77(h + 7) = 1001$

$\Rightarrow \quad h + 7 = 13$

$\Rightarrow \quad h = 6\text{cm}$

Height of toy = h + 3.5 = 6cm + 3.5cm = 9.5cm

Surface area of hemisphere = $2\pi r^2 = 2\pi(3.5)^2$

$$= 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 cm^2$$

$\therefore$ Cost = ₹ 10 × 77 = ₹770

35. First we represent the data in grouped frequency distribution table.

Marks (C.I.)	x_i	**Number of students (f_i)**	$u_i = \frac{x_i - 55}{10}$	f_iu_i
0 – 10	5	3	-5	-15
10 – 20	15	5	-4	-20
20 – 30	25	7	-3	-21
30 – 40	35	12	-2	-24
40 – 50	45	10	-1	-10
50 – 60	55 = A	20	0	0
60 – 70	65	7	1	7
70 – 80	75	6	2	12
80 – 90	85	2	3	6
90 – 100	95	8	4	32
		$\Sigma f_i = 80$		$\Sigma f_iu_i = -33$

Mean = $A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$

$= 55 - \frac{33}{80} \times 10$ = 55 – 4.125 = 50.875 marks

36. **(i)** In ΔPAB and ΔPQR,

$\angle P = \angle P$ (Common)

$\angle A = \angle Q$ (Corresponding angles)

By AA similarity criterion, $\Delta PAB \sim \Delta PQR$

$$\therefore \frac{AB}{QR} = \frac{PA}{PQ} \Rightarrow \frac{AB}{12} = \frac{6}{24} \Rightarrow AB = 3m$$

(ii) Similarly, ΔPCD and ΔPQR are similar.

$$\therefore \frac{PC}{PQ} = \frac{CD}{QR} \Rightarrow \frac{14}{24} = \frac{CD}{12} \Rightarrow CD = 7m$$

(iii) (a) From the figure, by Basic Proportionality theorem, we have

In ΔPAB and ΔPQR

$$\frac{PA}{PQ} = \frac{PB}{PR} \quad (\because AB \parallel QR)$$

$$\Rightarrow \frac{PB}{PR} = \frac{6}{24} \Rightarrow \frac{PB}{PR} = \frac{1}{4}$$

OR

(iii) (b) As from the figure, in ΔPAB and ΔPCD,

By using Basic Proportionality theorem, we have

$$\frac{PB}{PD} = \frac{PA}{PC} \quad (\because AB \parallel CD)$$

$$\Rightarrow \frac{PB}{PD} = \frac{6}{14} \Rightarrow \frac{BP}{PD} = \frac{3}{7}$$

37. **(i)** The outer surface area of hemisphere = $2\pi r^2$ ($\because$ r = 20cm)

$$= 2 \times \frac{22}{7} \times 20 \times 20 = \frac{17600}{7} = 2514.28 cm^2$$

(ii) We have, r = 20cm, h = 30cm

Total surface area of type (I) container

= $2\pi rh + \pi r^2 + 2\pi r^2$

$= \frac{22}{7} \times 20[60 + 60] = \frac{22 \times 20 \times 120}{7} = \frac{52800}{7}$

= 7542.86 cm^2

(iii) (a) Volume of type (I) container

$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{2}{3} r\right)$

$= \frac{22}{7} \times 20 \times 20 \left[30 + \frac{2}{3}(20)\right] = \frac{8800}{7} \times \frac{130}{3} = \frac{1144000}{21}$ = 54476.19cm^3

OR

(iii) (b) We have, r = 20cm

Volume of type (II) container $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 20 \times 20 \times 20 = \frac{352000}{21} = 16761.90 cm^3$

38. **(i)** Number of throws during 11th day of camp is given by

$t_{11} = a + 10d$ $[\because a = 40 \text{ and } d = 12]$

= 40 + 10 × 12 = 160 throws

(ii) a = 40; d = 12 and n = 15

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[2(40) + (15-1)(12)] = \frac{15}{2}[80 + 168] \quad = \frac{15}{2}[248] = 1860$ throws

(iii) (a) Given, a = 7.56 m; d = 9cm = 0.09m and n = 6 weeks

$\therefore t_n = a + (n-1)d$

= 7.56 + 5(0.09) = 7.56 + 0.45 = 8.01m

$\therefore$ Sanitha's throw distance at the end of 6 weeks = 8.1 m

OR

(iii) (b) a = 7.56 m; d = 9cm = 0.09m and t_n = 11.16m

$\because t_n = a + (n-1)d$

$\Rightarrow$ 11.16 = 7.56 + (n – 1)(0.09) $\Rightarrow$ 3.6 = (n – 1)(0.09)

$\Rightarrow n - 1 = \frac{3.6}{0.09} = 40 \Rightarrow n = 41$

Sanjitha's will be able to throw 11.16m in 41 weeks.

SOLUTIONS (Sample Paper – 5)

1. **(c)** As, $HCF \times LCM = 24 \times 32$

$\Rightarrow$ $HCF \times LCM = 768$

2. **(b)** 1, as it cuts the x-axis at one point.

3. **(d)** The given equations are:

$$5x - 6y - 7 = 0 \text{ and } -15x + \alpha y + 21 = 0$$

For infinitely many solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{5}{-15} = \frac{-6}{\alpha} = \frac{-7}{21}$$

$$\Rightarrow \quad \alpha = 18$$

4. **(a)** We have, $D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

So, given quadratic equation has no real roots.

5. **(d)** AP is $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}.....$, Here $a = \sqrt{2}, d = \sqrt{2}$

As, $a_n = a + (n - 1)d$

So, $a_7 = a + 6d$

$= \sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$

6. **(a)**

Let P and Q be the points of trisection.

Now, Q divides AB in the ratio of 2 : 1.

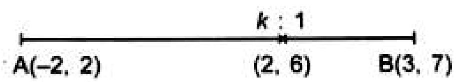
By section formula,

$$\alpha = \frac{2 \times 0 + 1 \times (-1)}{2+1}, \beta = \frac{2 \times 4 + 1 \times 3}{2+1}$$

So, $\alpha = \frac{-1}{3}, \beta = \frac{11}{3}$

Coordinates of the required point are $\left(\frac{-1}{3}, \frac{11}{3}\right)$.

7. **(b)**



Let ratio be k : 1.

$$\left(\frac{3k-2}{k+1}, \frac{7k+2}{k+1}\right) = (2,6)$$

So, $\frac{3k-2}{k+1} = 2 \Rightarrow 3k - 2 = 2k + 2 \Rightarrow k = 4$

and $\frac{7k+2}{k+1} = 6 \Rightarrow 7k + 2 = 6k + 6 \Rightarrow k = 4$

So, ratio is 4 : 1.

8. **(b)**

9. **(d)** AR = 4cm, RB = 3 c, AC = 11 cm

As, tangents drawn from an external point to a circle are equal. Then

$AQ = AR \Rightarrow AQ = 4$ cm

Now, $QC = 11$ cm – 4cm = 7 cm

Now, $PC = QC = 7$ cm
and $BP = RB = 3$ cm
$\therefore$ $BC = 3\text{ cm} + 7\text{ cm} = 10$ cm

10. **(a)** Tangents drawn from an external point to a circle are equal in length. Then,

$AQ = AR$, $BQ = BX$ and $CR = CX$

Now, perimeter of $\Delta ABC = AB + BC + CA$

$\Rightarrow$ $12 = AQ - BQ + BX + XC + AR - CR$

$\Rightarrow$ $12 = AR - BQ + BQ + CR + AR - CR$

$\Rightarrow$ $12 = 2AR$

$\Rightarrow$ $AR = 6$ cm

11. **(b)** $\sin\theta = \frac{a}{b}$

Now, $\cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\frac{a^2}{b^2}}$ $= \frac{\sqrt{b^2-a^2}}{b}$

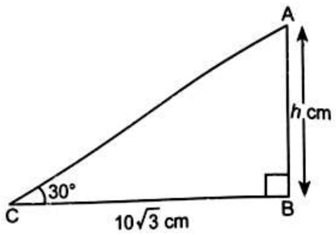
So, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{b^2-a^2}}{a}$

12. **(a)** $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = k$

$\Rightarrow$ $\tan^2\theta \cdot \cot^2\theta = k$

$\Rightarrow$ $1 = k$ or $k = 1$

13. **(b)** In ΔABC, $\frac{h}{10\sqrt{3}} = \tan 30^\circ$



$\Rightarrow$ $h = \frac{10\sqrt{3}}{\sqrt{3}} = 10\text{cm}$

14. **(c)** Perimeter = $\frac{270^\circ}{360^\circ} \times 2\pi(42) + 2 \times 42$

$= \frac{3}{4} \times 2 \times \frac{22}{7} \times 42 + 84$ = 198 cm + 84 cm = 282 cm

15. **(c)** Let r be the radius,

ATQ, $\frac{\theta}{360^\circ} \times 2\pi r = 2\pi r \times \frac{7}{9}$ [θ = angle subtended by major arc at the centre]

$\Rightarrow$ $\theta = 280^\circ$

So, angle subtended by minor arc at centre = $360^\circ - 280^\circ = 80^\circ$

16. **(b)** P(picks up a red flower) = P(number $r \le 4$) = $\frac{4}{6} = \frac{2}{3}$

17. **(a)** When a coin is tossed thrice, then possible outcomes are :

{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

Let E be the event of getting at least 2 heads. Outcomes favourable to E are :

{HHT, HTH, THH, HHH}

$\therefore \quad P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcome}} = \dfrac{4}{8} = \dfrac{1}{2}$

18. **(c)** Let mode be 3x and mean be x

Now, mode = 3 median – 2 mean

$\Rightarrow$ 3x = 3 median – 2x

$\Rightarrow$ 3 median = 5x

$\Rightarrow$ 3 × median = 5 × mean

$\Rightarrow$ mean : median = 3 : 5

19. **(a)** Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. **(d)** Assertion (A) is false but reason (R) is true.

21. Let if possible, $\sqrt{11}$ is a rational number.

Then, $\sqrt{11} = \dfrac{a}{b}$, where a, b are coprime integers and $b \neq 0$

$\Rightarrow \quad 11 = \dfrac{a^2}{b^2} \Rightarrow a^2 = 11b^2$ .....(i)

$\Rightarrow$ a^2 is a multiple of 11

$\Rightarrow$ a is a multiple of 11

$\Rightarrow$ a = 11 m, where m is any integer. .....(ii)

Substituting a = 11 m in (i), we get

$(11m)^2 = 11b^2 \Rightarrow b^2 = 11\,m^2$

$\Rightarrow$ b^2 is a multiple of 11

$\Rightarrow$ b is a multiple of 11

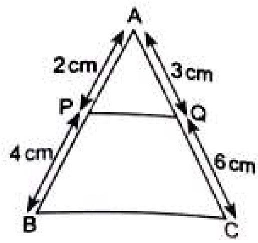
$\Rightarrow$ b = 11n, where n is any integer.

From (ii) and (iii), we get that a and b are not coprime. as 11 is the common factor of a and b.

So, this is a contradiction. So, our assmuption is wrong.

Hence, $\sqrt{11}$ is an irrational number.

22. We have, $\dfrac{AP}{PB} = \dfrac{1}{2}, \dfrac{AQ}{QC} = \dfrac{1}{2}$



$\Rightarrow$ PQ || BC (Converse of BPT)

In ΔAPQ and ΔABC,

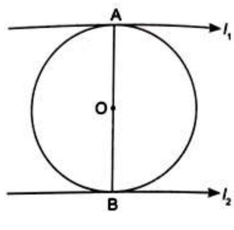
$\angle APQ = \angle ABC$ (Corresponding angles)

$\angle AQP = \angle ACB$ (Corresponding angles)

$\therefore$ $\Delta APQ \sim \Delta ABC$ (AA similarity)

$\therefore \quad \dfrac{AP}{AB} = \dfrac{PQ}{BC} \Rightarrow \dfrac{2}{6} = \dfrac{PQ}{BC} \Rightarrow BC = 3PQ$

23. Let AB be diameter of a circle with centre O. suppose l_1 and l_2 are tangents to circle at A and B respectively.



As, tangent to a circle is always perpendicular to its radius at the point of contact.

So, $OA \perp l_1$ or $BA \perp l_1$ .....(i)

and $OB \perp l_2$ or $AB \perp l_2$ .....(ii)

Two lines perpendicular to same line are parallel to each other.

$\therefore$ From (i) and (ii), we get $l_1 \parallel l_2$.

24. We have, AB = x units, AC = 7 units and $\angle B = 90^\circ$

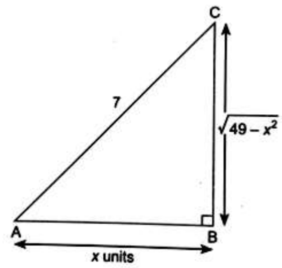
$\therefore \quad BC = \sqrt{49 - x^2}$

Now, $\sqrt{7-x}\tan C + \sqrt{7+x}\cot A - 14\cos A + 21\sin C + \sqrt{49+x^2}\cos B$

$$= \sqrt{7-x} \times \frac{x}{\sqrt{49-x^2}} + \sqrt{7+x} \times \frac{x}{\sqrt{49-x^2}} - 14 \times \frac{x}{7} + 21 \times \frac{x}{7} + \sqrt{49+x^2} \times \cos 90^o$$

$$= \frac{x}{\sqrt{49-x^2}}\left(\sqrt{7-x} + \sqrt{7+x}\right) + 7.\frac{x}{7} + 0$$

$$= x\left[\frac{\sqrt{7-x} + \sqrt{7+x} + \sqrt{49-x^2}}{\sqrt{49-x^2}}\right]$$



OR

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\sec\theta\sin\theta} = \frac{1}{xy}$$

Now, $\tan\theta = \frac{1}{\cot\theta} = xy$

$$\frac{\cot\theta + \tan\theta}{\cot\theta - \tan\theta} = \frac{\frac{1}{xy} + xy}{\frac{1}{xy} - xy} = \frac{1 + x^2y^2}{1 - x^2y^2}$$

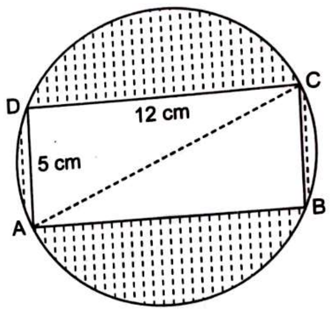
25. Area of shaded region = Area of larger sector – Area of smaller sector

$$= \frac{30^o}{360^o}\pi[(7)^2 - (3.5)^2]$$

$$= \frac{1}{12}\pi(49 - 12.25) = \frac{1}{12} \times \frac{22}{7} \times 36.75 \qquad = 6.625 \text{ cm}^2$$

OR

Join AC.



Diagonal of rectangle ABCD = Diameter of circle

$$= \sqrt{12^2 + 5^2} = 13 \text{ cm}$$

$\therefore$ Area of shaded region = area of circle – area of rectangle ABCD

$$= \pi\left(\frac{13}{2}\right)^2 - 12 \times 5$$

$= 3.14 \times (6.5)^2 - 60$

$= 132.67 - 60 = 72.67 \text{ cm}^2$

26.

Subject	**Mathematics**	**Science**	**Social Science**
Number of books	28	16	12

(a) We have to find HCF of 28, 16, 12

Now, $28 = 2^2 \times 7$

$16 = 2^4$

$12 = 2^2 \times 3$

$\therefore$ HCF $= 2^2 = 4$

So, each student got 4 books.

(b) Total number of students who got the books = $\left(\frac{28}{4}+\frac{16}{4}+\frac{12}{4}\right)$

$= 7 + 4 + 3 = 14$

27. As, α and β are zeroes of the polynomial $3x^2 + 2x - 1$,

$\therefore \quad \alpha + \beta = -\frac{2}{3}, \alpha\beta = -\frac{1}{3}$

Now, zeroes of required polynomial are $2\alpha + 1$ and $2\beta + 1$

Sum, $\quad S = 2\alpha + 1 + 2\beta + 1 = 2(\alpha + \beta + 1)$

$= 2\left(-\frac{2}{3}+1\right) = 2\times\frac{1}{3} = \frac{2}{3}$

Product, $\quad P = (2\alpha + 1)(2\beta + 1)$

$= 4\alpha\beta + 2\alpha + 2\beta + 1$

$= -\frac{4}{3} - \frac{4}{3} + 1 = -\frac{8}{3} + 1 = -\frac{5}{3}$

$\therefore$ Required polynomial = $k[x^2 - Sx + P]$, k is non-zero real number.

$= k\left[x^2 - \frac{2x}{3} - \frac{5}{3}\right]$

$= \frac{k}{3}(3x^2 - 2x - 5)$

$= 3x^2 - 2x - 5$ (Taking k = 3)

28. Let the digit at one's place and ten's place be y and x respectively.

So, original number = 10x + y

ATQ, $\quad \frac{10x+y}{x+y} = 7 \Rightarrow 10x + y = 7x + 7y$

$\Rightarrow \quad 3x = 6y \Rightarrow x = 2y$ …..(i)

Also, $\quad 10x + y - 27 = 10y + x \Rightarrow 9x - 9y = 27$

$\Rightarrow \quad x - y = 3$ [From (i)]

$\Rightarrow \quad 2y - y = 3$

$\Rightarrow \quad y = 3$

From (i), $\quad x = 2 \times 3 = 6$

$\therefore$ Number = 60 + 3 = 63

OR

We have,

$5x + \frac{4}{y} = 9$ ……(i)

$7x - \frac{2}{y} = 5$ …..(ii)

Multiplying (ii) by '2', we get

$14x - \frac{4}{y} = 10$ …..(iii)

Adding (i) and (iii), we get

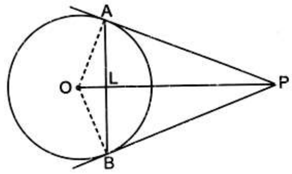
$19x = 19 \Rightarrow x = 1$

$\therefore$ From (i),

$5 + \frac{4}{y} = 9 \quad \Rightarrow \quad \frac{4}{y} = 4 \Rightarrow \quad y = 1$

$\therefore$ Solution is x = 1, y = 1

29. Given : PA and PB are tangents to the circle with centre O, with points of contact A and B respectively.



To prove : OP is right bisector of AB.

Construction : Join OA and OB.

Proof : Consider ΔOAP and ΔOBP,

OA = OB (Radii)

AP = BP

(Tangents drawn from an external point to a circle are equal)

OP = OP (Common)

$\therefore$ $\Delta OAP \cong \Delta OBP$ [SSS congruency]

$\Rightarrow$ $\angle AOP = \angle BOP$ or $\angle AOL = \angle BOL$ [CPCT] …….(i)

Consider Δs OAL and OBL,

OA = OB (Radii)

OL = OL (Common)

$\angle AOL = \angle BOL$ [from (i)]

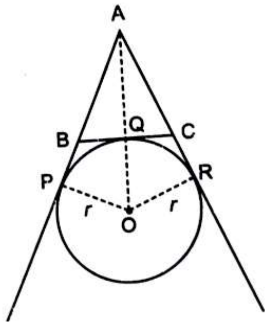
$\therefore$ $\Delta AOL \cong \Delta BOL$ (SAS congruency)

$\Rightarrow$ AL = BL [CPCT] and $\angle OLA = \angle OLB$ (CPCT)

Also, $\angle OLA = \angle OLB = 90^\circ$ [As $\angle ALO + \angle BLO = 180^\circ$]

$\therefore$ OL or OP is right bisector of AB.

OR



Given : Side, BC, touch the circle with centre O and radius 'r' at Q.

Sides AB and CA touch the circle when produced at points P and R respectively.

To prove : (a) AB + BQ = AC + CQ

(b) ar(APOR) = $\frac{1}{2}$(perimeter of $\Delta ABC) \times r$

Proof : As, lengths of tangents drawn from an external point to a circle are

So, AP = AR, BQ = BP, CQ = CR

(a) As, AP = AR

$\Rightarrow$ AB + BP = AC + CR

$\Rightarrow$ AB + BQ = AC + CQ [$\because$ BP = BQ and CR = CQ]

(b) As, tangent to a circle is perpendicular to radius at the point of contact.

So, $\angle OPA = \angle ORA = 90^\circ$

Now, ar(□APOR) = ar(ΔOPA) + ar(ΔORA)

$$= \frac{1}{2} \times AP \times OP + \frac{1}{2} \times AR \times OR$$

$$= \frac{1}{2} \times r \times (AP + AR)$$

$$= \frac{1}{2} \times r \times 2AP \qquad [\because AP = AR] \qquad .....(i)$$

Now, perimeter of ΔABC = AB + BC + CA

= AP – BP + BQ + QC + AR – CR

= AP – BP + BP + CR + AP – CR …..(ii)

= 2AP

$\therefore$ From (i) and (ii), we get

ar(quad. APOR) = $= \frac{1}{2} \times r \times$ perimeter of ΔABC

30. Consider $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$\Rightarrow$ $\frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{1}{\sin\theta} + \frac{1}{\sin\theta} = \frac{2}{\sin\theta}$

For proving the given identity, it is just sufficient to prove

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{2}{\sin\theta}$$

$$\text{LHS} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta}{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} \qquad = \frac{2\operatorname{cosec}\theta}{1} = \frac{2}{\sin\theta} = \text{RHS}$$

$\therefore$ $= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

31. First we represent data in continuous grouped frequency distribution table as :

Wages in (Rs.) C.I.	**f**	**c.f.**
80-90	9	9
90-100	17	26
100-110	19	45
110-120	45	90
120-130	33	123
130-140	15	138
140-150	12	150
	N = 150	

$\leftarrow$ Median class

$$\frac{N}{2} = \frac{150}{2} = 75$$

Median class : 110 – 120

So, l = 11, f = 45, cf = 45, h = 10

$$\text{Now, median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 110 + \frac{75 - 45}{45} \times 10 \qquad = 110 + \frac{30}{45} \times 10 = 110 + 6.67 = \text{Rs.}116.67$$

32. Let the total number of students who planned for picnic be x.

Cost of each student originally = $\text{Rs.}\frac{\text{total budget}}{x} = \text{Rs.}\left(\frac{1800}{x}\right)$

Number of students who attended picnic = (x – 4)

Now, cost to each student = $\text{Rs.}\left(\frac{1800}{x-4}\right)$

ATQ, $\frac{1800}{x-4} - \frac{1800}{x} = 5$

$\Rightarrow \quad \frac{1}{x-4} - \frac{1}{x} = \frac{1}{360} \quad \Rightarrow \quad \frac{x-x+4}{x(x-4)} = \frac{1}{360}$

$\Rightarrow \quad x^2 - 4x - 1440 = 0$

$\Rightarrow \quad x^2 - 40x + 36x - 1440 = 0$

$\Rightarrow \quad x(x - 40) + 36(x - 40) = 0$

$\Rightarrow \quad (x - 40)(x + 36) = 0$

$\Rightarrow \quad x = 40$ or $x = -36$ (rejected)

So, number of students who attend picnic = 40 – 4 = 36

Cost of each student = $\text{Rs.}\frac{1800}{36} = \text{Rs.}50$

OR

The given quadratic equation is,

$3x^2 + px - 8 = 0$

Since (-4) is a root of the above equation, then it must satisfy it. Then,

$3(-4)^2 + p \times (-4) - 8 = 0$

$\Rightarrow \quad 48 - 4p - 8 = 0$

$\Rightarrow \quad 4p = 40 \Rightarrow p = 10$

Here, the other equation is,

$px^2 + 3px - k = 0$

$\Rightarrow \quad 10x^2 + 30x - k = 0$

Now, $a = 10; b = 30; c = -k$

$D = b^2 - 4ac \quad = (30)^2 - 4 \times 10(-k) = 900 + 40k$

For equal roots, $D = 0$

$\Rightarrow \quad 900 + 40k = 0$

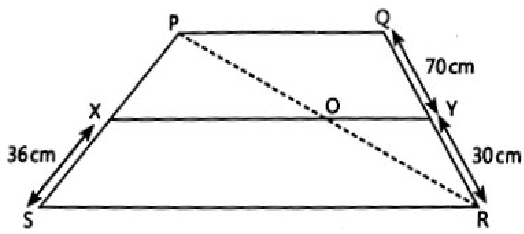
$\Rightarrow \quad k = \frac{-900}{40}$

$\Rightarrow \quad k = \frac{-45}{2}$

33. Join PR and suppose it intersects XY at O.

In ΔPSR, XO || SR, then

$\frac{PX}{XS} = \frac{PO}{OR}$ [BPT] .....(i)



Now, PQ || SR (Given) …..(ii)

Also XY|| SR or OY||SR …..(iii)

So, from (ii) and (iii), we get

OY || PQ

In ΔPRQ, OY || PQ

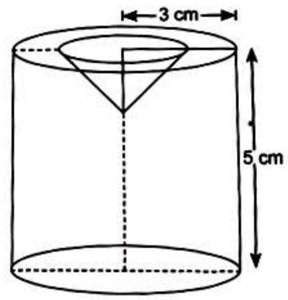
$\frac{RO}{OP} = \frac{RY}{YQ}$ [BPT]

$\Rightarrow \quad \frac{PO}{OR} = \frac{QY}{YR}$ ……(iv)

From (i) and (iv), we get

$$\frac{PX}{XS} = \frac{QY}{YR}$$

$\Rightarrow$ $$\frac{PX}{36} = \frac{70}{30} \Rightarrow PX = 84 \text{ cm}$$



34. For cylinder :

Radius of base, R = 3 cm

height, H = 5 cm

For cone :

Radius of base, $r = \frac{3}{2}$ cm

Height, $h = \frac{8}{9}$ cm

Volume of metal taken out, V_1 = Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 . \frac{8}{9} = \frac{2}{3}\pi \text{ cm}^3$$

Volume of metal left in cylinder, $V_2 = \pi R^2 H - \frac{1}{3}\pi r^2 h = \pi(3)^2 \times 5 - \frac{2}{3}\pi$

$\Rightarrow$ $$V_2 = 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3} \text{cm}^3$$

$\Rightarrow$ $V_2 : V_1 = 133 : 2$

Hence, volume of metal left in cylinder : Volume of metal taken out in conical shape = 133:2.

OR

Let height of rain water on roof = h m

Volume of water on roof

= lbh = 22 × 20 × h m^3 .....(i)

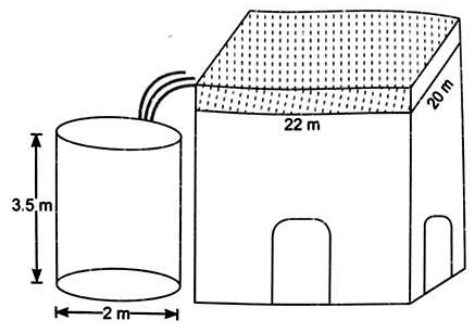
Volume of cylindrical vessel

$= \pi r^2 H = \pi(1)^3 \times 3.5 \text{ m}^3 = \frac{22}{7} \times \frac{35}{10} \text{m}^3$

As cylinder is filled with rainwater upto brim,

$\therefore$ $$22 \times 20 \times h = \frac{22}{7} \times \frac{35}{10}$$

$\Rightarrow$ $$h = \frac{22}{7} \times \frac{7}{2} \times \frac{1}{22 \times 20} \qquad = \frac{1}{40} m = 2.5 \text{ cm}$$



35. Median = 14.4 and $\sum f = N = 20$

C.I.	f	c.f.
0-6	4	4
6-12	x	4 + x
12-18	5	9 + x
18-24	y	9 + x + y
24-30	1	10 + x + y
	N = 150	

$\leftarrow$ Median class

Now, N = 20 $\Rightarrow$ 10 + x + y = 20 $\Rightarrow$ x + y = 10 ......(i)

Median class : 12 – 18

Now, l = 12, f = 5, cf = (4 + x), h = 6, N = 20

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$\Rightarrow$ $$14.4 = 12 + \frac{10-(4+x)}{5} \times 6 \Rightarrow 2.4 = \frac{(6-x)6}{5} \Rightarrow x = 4$$

From (i), we get y = 6

$\therefore$ x = 4, y = 6

Calculation of mode :

Modal class = 18 – 24

Here, l = 18, f_1 = 6, f_0 = 5, f_2 =1 and h = 6

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 18 + \frac{6-5}{12-5-1} \times 6 = 18 + \frac{1}{6} \times 6 = 19$$

36. (i) Since, every student get one chocolate.

So, number of chocolates Rohit has = Number of students in the class = 54.

Let number of milk chocolates Rohit has = x

Probability of distributing milk chocolates = $\frac{1}{3}$

$\Rightarrow \frac{x}{54} = \frac{1}{3} \Rightarrow x = \frac{54}{3} = 18$

(ii) Let number of dark chocolates Rohit has = y

Probability of distributing dark chocolates = $\frac{4}{9}$

$\Rightarrow \frac{y}{54} = \frac{4}{9} \Rightarrow \quad y = \frac{4 \times 54}{9} = 24$

(iii) (a) Number of white chocolates Rohit has

= 54 – (18 + 24) = 12

$\therefore$ Required probability = $\frac{12}{54} = \frac{2}{9}$

OR

(iii) (b) Now, number o milk chocolates Rohit has = 18

and number of white chocolates Rohit has = 12

$\therefore$ Required probability = $\frac{18+12}{54} = \frac{30}{54} = \frac{5}{9}$

37. **(i)** In ΔABC, $\frac{AB}{BC} = \tan 60^o$

$\Rightarrow AB = 25 \times \sqrt{3} = 25\sqrt{3}$

$\therefore$ Height of building is $25\sqrt{3}$m

(ii) In ΔABD, $\frac{AB}{BD} = \tan 30^o$

$\Rightarrow \frac{25\sqrt{3}}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 75\,m$

$\therefore$ Distance between two positions of car

= (75 – 25) m = 20 m

(iii) (a) Time taken to cover 50 m distance = 6 secs.

$\therefore$ Time taken to cover 25 m distance = 3 secs

$\therefore$ Total time taken by car = 6 secs + 3 secs = 9 secs

OR

(iii) (b) In ΔABD, $\frac{AB}{AD} = \sin 30^o$

$\Rightarrow \frac{25\sqrt{3}}{AD} = \frac{1}{2}$

$\Rightarrow AD = 50\sqrt{3}\text{ m} = 50 \times 1.732 = 86.6\text{ m}$

38. (i) Piyush sells a saree at 8% profit + sells a sweater at 10% discount = Rs. 1008

$\Rightarrow$ (100 + 8)% of x + (100 – 10)% of y = 1008

$\Rightarrow$ 108% of x + 90% of y = 1008

$\Rightarrow$ 1.08x + 0.9 y = 1008 .....(i)

(ii) Piyush sold the saree at 10% profit + sold the sweater at 8% discount = Rs. 1028

$\Rightarrow$ (100 + 10)% of x + (100 – 8)% of y = 1028

$\Rightarrow$ 110% of x + 92% of y = 1028

$\Rightarrow$ 1.1x + 0.92y = 1028 .....(ii)

(iii) (a) At x –axis, y = 0

$\Rightarrow 1.08x = 1008 \Rightarrow x = \frac{1008}{1.08} = \frac{2800}{3}$

OR

(iii) (b) At y-axis, x = 0

$\Rightarrow 0.92y = 1028 \Rightarrow y = \frac{1028}{0.92} = \frac{25700}{23}$

SOLUTIONS (Sample Paper – 6)

1. **(a)** $70 - 5 = 65$

$125 - 8 = 117$

5	65
13	13
	1

3	117
3	39
13	13
	1

Prime factorisation of $65 = 5 \times 13$

and $117 = 3 \times 3 \times 13$

$\therefore$ HCF = 13

So, the largest number which divides 70 and 125 leaving remainder 5 and 8 is 13.

2. **(a)** Let the roots be α and $\frac{1}{\alpha}$.

$$\alpha \times \frac{1}{\alpha} = \frac{c}{a}$$

$\Rightarrow$ $1 = \frac{\lambda - 4}{4}$ $\Rightarrow$ $\lambda = 8$

3. **(c)** $f(x) = x^2 + px + q$

Since α and β are the zeroes of polynomial, then

$\alpha + \beta = -$ and $\alpha\beta = q$

If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the zeroes of polynomial then

Sum of zeroes = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-p}{q}$

Product of zeroes = $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{q}$

Polynomial with zeroes $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$k\left[x^2 - \left(\frac{-p}{q}\right)x + \frac{1}{q}\right]$, where k is a non-zero real number.

or $k\left[\frac{qx^2 + px + 1}{q}\right]$

or $k'(qx^2 + px + 1) = (qx^2 + px + 1)$ [on taking $k' = \frac{k}{q}$ and $k' = 1$]

4. **(d)**

5. **(d)** Points are $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$

Distance = $\sqrt{[(a\cos\theta + b\sin\theta) - 0]^2 + [0 - (a\sin\theta - b\cos\theta)]^2}$

$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + b^2\cos^2\theta + a^2\sin^2\theta - 2ba\cos\theta\sin\theta}$

$= \sqrt{a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)} = \sqrt{(a^2 + b^2)}$ units

6. **(d)** Intersecting at (a, b)

7. **(a)** $\sin 2A = 2 \sin A$

If $A = 0^\circ$

$\sin 2 \times 0^\circ = 2\sin 0^\circ$

$\Rightarrow$ $0^\circ = 0^\circ$ (true) $[\because \sin 0^\circ = 0]$

$\therefore$ $A = 0^\circ$

8. **(a)** Given, $x = a\cos\theta$

$y = b\sin\theta$

$\therefore$ $b^2x^2 + a^2y^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2$

$= b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$

$= b^2a^2(\cos^2\theta + \sin^2\theta) = b^2a^2 = a^2b^2$

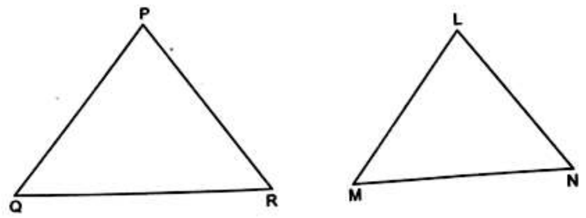
9. **(a)** As, $\Delta ABC \sim \Delta DEF$ (Given)

$\Rightarrow$ $\frac{AB}{DE} = \frac{BC}{EF}$ (Corresponding sides to similar triangles)

$\Rightarrow$ $\frac{AB}{2AB} = \frac{8}{EF}$ ($\because$ 2AB = DE)

$\Rightarrow$ EF = 16 cm

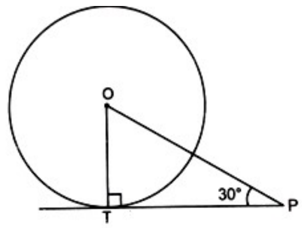
10. **(c)**



$\frac{PQ}{LM} = \frac{QR}{NL}$

So, $\angle Q = \angle L$ for $\Delta PQR \sim \Delta MLN$ by SAS similarity.

11. **(a)** We know that tangent is perpendicular to the radius at the point of contact.



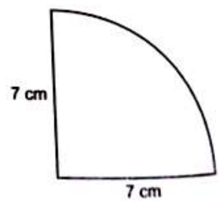
In ΔOTP, $\angle OTP + \angle OPT + \angle TOP = 180^\circ$

(By angle sum property of a triangle)

$\Rightarrow$ $90^\circ + 30^\circ + \angle TOP = 180^\circ$

$\Rightarrow$ $\angle TOP = 180^\circ - 120^\circ = 60^\circ$

12. **(d)** The diameter of quadrant of circle = $\frac{\pi r}{2} + 2r$



$= \frac{22}{7 \times 2} \times 7cm + 2 \times 7cm$

= 11 cm + 14 cm = 25 cm

13. **(a)** Capacity of tank = $\pi r^2 h$

$\Rightarrow$ $6160 = \frac{22}{7} \times 14 \times 14 \times h$

$\Rightarrow$ $h = \frac{6160 \times 7}{22 \times 14 \times 14} = 10m$

14. **(b)** Median

15. **(c)** $\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2}$ (Where C_1 and C_2 are circumference of two circles)

$\Rightarrow$ $\frac{r_1}{r_2} = \frac{2}{5}$

Now, $\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$ (Where A_1 and A_2 are areas of two circles)

16. **(c)** Since data is not continuous after making the continuous, the table will be

Weight (in g)	84.5-89.5	89.5-94.5	94.5-99.5	99.5-104.5	104.5-109.5
Number of eggs	10	12	15	4	2

Modal class = 94.5 – 99.5

Lower limit = 94.5

17. **(c)** $0 \leq P(A) \leq 1$

18. **(c)** sin (A + B) = 1 and cos (A – B) = 1 ……(i)

$\Rightarrow$ $A + B = 90^\circ$ …..(ii)

and $A - B = 0_o$

On adding both equations, we get

$A = 45^\circ$

From equation (ii),

$B = 45^\circ$

$\therefore$ $A = B = 45^\circ$

19. **(b)** LCM of 24, 36 and 12

LCM = 2 × 2 × 2 × 3 × 3 = 72

2	24, 36, 12
2	12, 18, 6
2	6, 9, 3
3	3, 9, 3
3	1, 3, 1
	1, 1, 1

72 is exactly divisible by 24, 36 and 12. So 75 will given remainder 3 when divided by 24, 36 and 12.

$\therefore$ Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

20. **(a)** Let point P(x, y) divides AB in the ratio 2 : 3.

Coordiantes are $\left(\frac{12}{5}, 1\right)$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

21. Let numbers be x and y

According to the question

$x + y = 12$ …..(i)

and $x = 2y - 3$ …..(ii)

Putting the value of x from (ii) and (i), we get

$2y - 3 + y = 12$

$\Rightarrow$ $3y = 15 \Rightarrow y = 5$

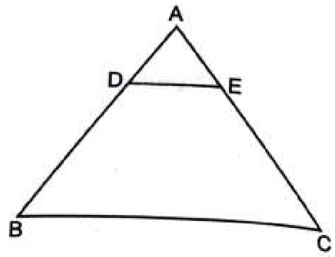
Putting the value of y in (ii), we get

$x = 7$

Hence, numbers are 7 an 5.

22. Given in ΔABC, DE || BC

So, $\frac{AD}{DB} = \frac{AE}{EC}$ (By BPT)



$\Rightarrow \frac{3}{5} = \frac{AE}{AC - AE}$

$\Rightarrow \frac{3}{5} = \frac{AE}{4.8 - AE}$

$\Rightarrow 14.4 - 3AE = 5AE$

$\Rightarrow 8AE = 14.4$

$\Rightarrow AE = \frac{14.4}{8} cm = 1.8 cm$

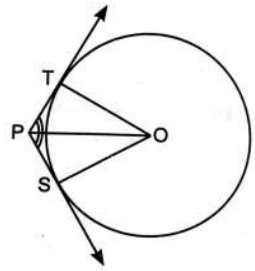
23. $\angle SPT = 120^o$

As, tangents drawn from an external point to a circle are always equally inclined to the segment joining centre to that point.'

So, $\angle OPS = 60^o$

In ΔOPS, $\angle OSP = 90^o$

[Tangent to a circle is perpendicular to its radius drawn through point of



Now, $\cos(\angle OPS) = \frac{PS}{OP}$

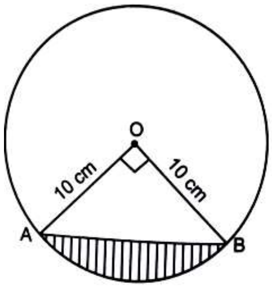
$\Rightarrow \cos 60^o = \frac{PS}{OP}$

$\Rightarrow \frac{1}{2} = \frac{PS}{OP}$

$\Rightarrow OP = 2PS$

24. Area of minor segment = area of sector – area of ΔAOB

$$\frac{\pi r^2 \theta}{360^o} - \frac{1}{2} \times 10 \times 10$$



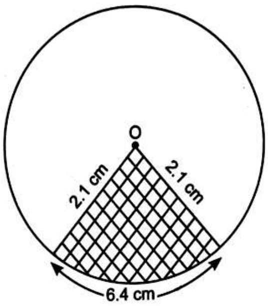
$$= \frac{22}{7} \times \frac{100 \times 90^o}{360^o} - \frac{1}{2} \times 100$$

$$= \frac{22 \times 25}{7} - 50 = \frac{550 - 350}{7} = \frac{200}{7} cm^2$$

OR

Perimeter of sector = 6.4 cm

$\Rightarrow \frac{\pi r \theta}{180^o} + 2r = 6.4$



$\Rightarrow \frac{22}{7} \times 2.1 \times \frac{\theta}{180^o} + 2 \times 2.1 = 6.4$

$\Rightarrow 22 \times 0.3 \times \frac{\theta}{180^o} = 6.4 - 4.2$

$\Rightarrow \theta = \frac{2.2 \times 180^o}{22 \times 0.3} = 60^o$

Area of sector = $\frac{\pi r^2 \theta}{360^o}$

$$= \frac{22}{7} \times 2.1 \times 2.1 \times \frac{60^o}{360^o} = 2.31 \text{ cm}^2$$

25. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \times \tan B}$

$$\tan(A+B) = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}$$

$\Rightarrow$ $$\tan(A+B) = \frac{\frac{5}{6}}{1-\frac{1}{6}} = 1 = \tan 45^o$$

$\therefore$ $A + B = 45^o$

OR

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

LHS = $\cos 2A = \cos 2 \times 30^o = \cos 60^o = \frac{1}{2}$ $[\because A = 30^o]$

RHS = $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \dfrac{1-\tan^2 30^o}{1+\tan^2 30^o} = \dfrac{1-\frac{1}{3}}{1+\frac{1}{3}} = \dfrac{\frac{2}{3}}{\frac{4}{5}} = \dfrac{1}{2}$

$\therefore$ LHS = RHS Hence proved

26. Let $\sqrt{2}$ is a irrational number.

i.e., $\sqrt{2} = \dfrac{p}{q}$, where p and q are coprime and $q \neq 0$

$\Rightarrow$ $p = \sqrt{2}q$

On squaring both sides, we get ……(i)

$p^2 = 2q^2$

$\therefore$ 2 is the factor of p^2.

$\Rightarrow$ 2 is the factor of p.

Let p = 2k, where k is an integer

27. Given, $\alpha + \beta = 24$

$\alpha - \beta = 8$

Now, $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$\Rightarrow$ $(24)^2 = 8^2 + 4\alpha\beta$

$\Rightarrow$ $576 = 64 + 4\alpha\beta$

$\Rightarrow$ $\dfrac{512}{4} = \alpha\beta \Rightarrow \alpha\beta = 128$

$P(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

Polynomial is $k(x^2 - 24x + 128)$, where k is non-zero real number.

$\therefore$ Polynomial = $x^2 - 24x + 128$ (Taking k = 1)

28. Let fixed charges be Rs. X and charges per km be Rs. Y

According to the question

$x + 10y = 105$ ……(i)

and $x + 15y = 155$ ….(ii)

On solving (i) and (ii), we get

$y = 10$ and $x = 5$

Hence, fixed charge = Rs 5

Charge per km = Rs. 10

Charge for 25 km = x + 25y = Rs. 5 + Rs. 25 × 10 = Rs. 5 + Rs. 250 = Rs. 255

OR

Let the amount of money with A and B be Rs. x and Rs.y respectively.
If A gives Rs. 20 to B, then

$$\frac{3}{2}(x-20)=y+20$$

$\Rightarrow$ $3x - 60 = 2y + 40$

$\Rightarrow$ $3x - 2y = 100$ .....(i)

If B gives Rs. 50 to A, then

$x + 50 = 2(y - 50) + 10$

$\Rightarrow$ $x + 50 = 2y - 100 + 10$

$\Rightarrow$ $x - 2y = -140$ .....(ii)

Subtracting equation (i) from (ii), we get

$x - 2y = -140$ .....(ii)

$3x - 2y = 100$ .....(i)

$-\quad +\quad -$

$-2x = -240$ $\Rightarrow$ $x = 120$

Putting value of x in (ii), we get

$120 - 2y = -140$

$\Rightarrow$ $-2y = -260$

$\Rightarrow$ $y = 130$

So A has Rs. 120 and B has Rs. 130.

29. $$\frac{\sin A}{\cot A+\operatorname{cosec}A}=2+\frac{\sin A}{\cot A-\operatorname{cosec}A}$$

$$\frac{\sin A}{\cot A+\operatorname{cosec}A}-\frac{\sin A}{\cot A-\operatorname{cosec}A}=2$$

$$\text{LHS}=\sin A\left[\frac{1}{\cot A+\operatorname{cosec}A}-\frac{1}{\cot A-\operatorname{cosec}A}\right]=\frac{\sin A(\cot A-\operatorname{cosec}A-\cot A-\operatorname{cosec}A)}{(\cot A+\operatorname{cosec}A)(\cot A-\operatorname{cosec}A)}$$

$$=\sin A\left[\frac{-2\operatorname{cosec}A}{\cot^2 A-\operatorname{cosec}^2 A}\right]$$

$$=\frac{\sin A\left[\frac{-2}{\sin A}\right]}{-1(\operatorname{cosec}^2 A-\cot^2 A)}\qquad\left[\because \operatorname{cosec}A=\frac{1}{\sin A}\right]$$

$= 2 = \text{RHS}$ $[\operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$$\therefore\quad \frac{\sin A}{\cot A+\operatorname{cosec}A}=2+\frac{\sin A}{\cot A-\operatorname{cosec}A}$$

30. Let OA = OT = r cm

In ΔOTB,

$\angle OTB = 90^o$

(Radius drawn through point of contact is perpendicular to tangent)

So $OB^2 = OT^2 + TB^2$ ($\because$ Pythagoras Theorem)

$\Rightarrow$ $(r + 24)^2 = r^2 + 36^2$

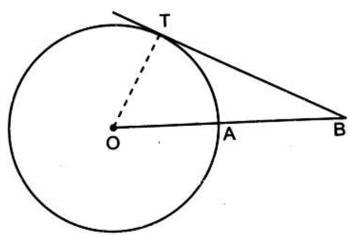
$\Rightarrow$ $r^2 + 24^2 + 48r = r^2 + 36^2$

$\Rightarrow$ $48r = 36^2 - 24^2$

$\Rightarrow$ $48r = (36 + 24)(36 - 24)$

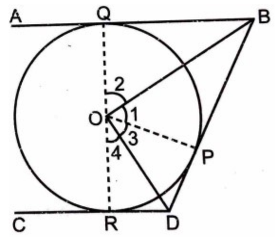
$$r=\frac{60\times 12}{48}$$

$r = 15$ cm



OR

Given : AB and CD are two tangents to a circle with centre O and AB||CD.
Tangent BD students ∠BOD at the centre.



To prove : $\angle BOD = 90^o$.
Construction : Join OQ, OP and QR.
Proof : OP ⊥ BD. [Tangent at any point of a circle is perpendicular to the radius through the point of contact.]
In right-angled Δs OQB and OPB,

OQ = OP
BQ = BP [Tangents drawn from an external point to a circle are equal]
OB = OB

⇒ $\Delta OQB \cong \Delta OPB$

⇒ $\angle 2 = \angle 1$

Similarly in right-angled Δs OPD and ORD

$\angle 3 = \angle 4$

∴ $\angle BOD = \angle 1 + \angle 3$

$$= \frac{1}{2}[2\angle 1 + 2\angle 3]$$

$$= \frac{1}{2}(\angle 1 + \angle 1 + \angle 3 + \angle 3)$$

$$= \frac{1}{2}(\angle 1 + \angle 2 + \angle 3 + \angle 4) \quad [\because \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$$

$$= \frac{1}{2}(180^o) = 90^o.$$

[As QOR is a diameter of a circle and sum of all angles at a point on a line is 180°]

31. (i) Number divisible by 2 or 3 = numbers divisible by 2 + numbers divisible by 3 – numbers divisible by 6

= 10 + 6 – 3
= 13

P(number divisible by 2 or 3) = $\frac{13}{20}$

(ii) Numbers divisible by 2 and 3 = Numbers divisible by 6 i.e., 6, 12, 18

P(number divisible by 2 and 3) = $\frac{3}{20}$.

(iii) Two-digit numbers divisible by 3 are 12, 15, 18, i.e., 3

P(a two-digit number divisible by 3) = $\frac{3}{20}$

32. Let the usual speed of plane be x km/h
Increased speed = (x + 250) km/h
According to the question

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$$

⇒ $$\frac{1500(x + 250 - x)}{x(x+250)} = \frac{1}{2}$$

⇒ $1500 \times 250 \times 2 = x^2 + 250x$

⇒ $x^2 + 250x - 750000 = 0$

Discriminant, $D = b^2 - 4ac$

$= (250)^2 - 4 \times 1 \times (-750000)$

$= 62500 \pm 3000000 = 3062500$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-250 \pm \sqrt{3062500}}{2 \times 1} = \frac{-250 \pm 1750}{2}$$

$$= \frac{-250 + 1750}{2}, \frac{-250 - 1750}{2}$$

Taking positive sign because speed cannot be negative,

$$x = \frac{-250 + 1750}{2}$$

$\Rightarrow$ $x = 750$

Usual speed of plane = 750 km/h

OR

Let the time taken to fill tank tap with smaller diameter be x hours.

Time taken to fill tank by tap with larger diameter be (x – 2) hours.

Tank filled by tap with smaller diameter in 1 hour = $\frac{1}{x}$ part

Tank filled by tap with larger diameter in 1 hour = $\frac{1}{x-2}$ part

According to the question

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$\Rightarrow$ $\frac{x-2+x}{x(x-2)} = \frac{8}{15} \Rightarrow \frac{2x-2}{x^2-2x} = \frac{8}{15}$

$8x^2 - 16x = 30x - 30$

$\Rightarrow$ $8x^2 - 46x + 30 = 0$

$\Rightarrow$ $4x^2 - 23x + 15 = 0$

$\Rightarrow$ $(x - 5)(4x - 3) = 0$

$\Rightarrow$ $(x - 5) = 0$ or $4x - 3 = 0$

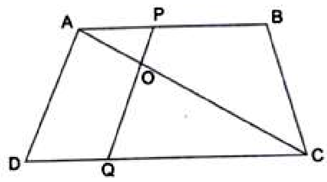
$\Rightarrow$ $x = 5$ or $x = \frac{3}{4}$

$\therefore$ $x - 2 = 3$ or $x - 2 = \frac{-5}{4}$ (rejected)

Time taken to fill tank by the tap with smaller diameter = 5 hours

Time taken to fill tank by the tap with larger diameter = 3 hours

33. **Given :** AB||DC and AC and PQ intersect at O.



To prove : OA.CQ = OC.AP

Proof : In ΔAOP and ΔCOQ

$\angle AOP = \angle COQ$ (Vertically opposite angles)

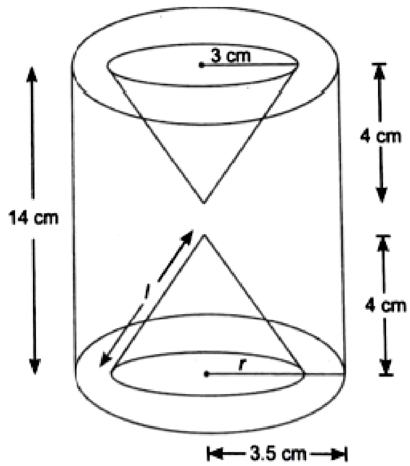
$\angle APO = \angle CQO$ and $\angle PAO = \angle OCQ$ (Alternating interior angles)

$\Delta AOP \sim \Delta COQ$ (By AAA criteria)

So, $\frac{AO}{CO} = \frac{AP}{CQ}$ [Corresponding sides of similar triangles]

$\Rightarrow$ AO.CQ = CO.AP Hence proved

34.



R = Radius of cylinder = 3.5 cm

r = Radius of cone = 3 cm

H = height of cylinder = 14 cm

h = height of cone = 4 cm

$l = \sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2} = 5\,\text{cm}$

Surface area of remaining solid = Curved surface area of cylinder + 2 × curved surface area of a cone + 2 × area of rings of remaining solid

$= 2\pi RH + 2\pi rl + 2(\pi R^2 - \pi r^2)$

$= 2\pi(RH + rl + R^2 - r^2)$

$= 2 \times \frac{22}{7}[3.5 \times 14 + 3 \times 5 + (3.5)^2 - 3^2]$

$= 2 \times \frac{22}{7}[49.0 + 15 + 12.25 - 9] = 2 \times \frac{22}{7} \times 67.25 = \frac{2959}{7}\text{cm}^2$

OR

Let n marbles are dropped into the beaker.

∴ Volume of n marbles = Volume of water raised

$n \times \frac{4}{3}\pi r^3 = \pi R^2 H$ [∵ r = radius of marbles, H = height of water level raised and R = radius of base of cylinder]

⇒ $n \times \frac{4}{3} \times (0.7)^3 = (3.5)^2 \times 5.6$

⇒ $n = \frac{3.5 \times 3.5 \times 5.6 \times 3}{4 \times 0.7 \times 0.7 \times 0.7}$

⇒ n = 150

35.

Length (in mm) (C.I.)	**Number of leaves (f)**	**Cumulative frequency (cf)**
117.5-126.5	5	5
126.5-135.5	6	11
135.5-144.5	10	21
144.5-153.5	13	34
153.5-162.5	6	40

$\frac{n}{2} = \frac{40}{2} = 20$

Median class = 135.5 – 144.5

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right) \times h}{f}$$

$$= 135.5 + \frac{(20-11)\times 9}{10} \qquad = 135.5 + \frac{9 \times 9}{10} \qquad = 135.5 + 8.1 = 143.6\text{ mm}$$

36. (i) Total distance covered by Dev watering 2nd tree and coming back

= 2 × 4 + 2 × 10

= 28 m

(ii) Distance covered by Dev watering 1st tree and coming back (a_1) = 8 m

Distance covered by Dev watering 2nd tree and coming back (a_2) = 20 m

Distance covered by Dev watering 3rd and coming back (a_3) = 32 m

$\therefore \quad d = a_2 - a_1 = a_3 - a_2 = 12$

Since, difference is common.

$\therefore$ Yes, obtained sequence is an AP.

(iii) $a_7 = a + 6d$

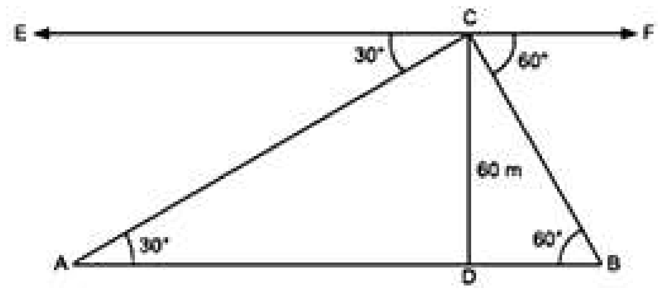
= 8 + 6 × (12) = 8 + 72 = 80 m

OR

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{20} = \frac{20}{2}[2 \times 8 + (20 - 1) \times 12] = 10(16 + 228) = 2440\text{ m}$

37.



As EF||AB

$\angle ECA = \angle CAD = 30^\circ$

and $\angle FCB = \angle CBD = 60^\circ$

(i) In ΔADC

$\Rightarrow \frac{DC}{AC} = \sin 30^\circ$

$\Rightarrow \frac{60}{AC} = \frac{1}{2}$

$\Rightarrow$ AC = 120 m

Hence, the distance of pravesh and boat when it is at point A is 120 m.

(ii) ΔBDC,

$\Rightarrow \frac{DC}{BC} = \sin 60^\circ$

$\Rightarrow \frac{60}{BC} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{3}BC = 120$

$\Rightarrow$ $BC = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 40\sqrt{3}m$

Hence, the distance of pravesh and boat when it is at point B is $40\sqrt{3}m$.

(iii) In ΔADC,

$$\frac{DC}{AD} = \tan 30^o$$

$\Rightarrow$ $$\frac{60}{AD} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $AD = 60\sqrt{3}m$ = 60 × 1.732m = 103.92 m

In ΔBDC,

$$\frac{DC}{BD} = \tan 60^o$$

$\Rightarrow$ $$\frac{60}{BD} = \sqrt{3}$$

$\Rightarrow$ $$\sqrt{3}BD = 60$$

$\Rightarrow$ $$BD = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}m = 20 \times 1.732m = 34.64m$$

$\therefore$ Width of river = AD + BD

= 103.92 m + 34.64 m

= 138.56 m

OR

Speed of boat = $\frac{\text{Distance}}{\text{Time}}$

$$= \frac{138.56}{15} = 9.24m/\min$$

38. (i) (5, 2)

(ii) (-5, 7)

(iii) Distance between Prime Minister's seat and opposition Minister's seat

$$= \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2} = \sqrt{(-5-5)^2 + (7-2)^2} = \sqrt{125} = 5\sqrt{5} \text{ units}$$

OR

Distance between origin and PM's seat = $\sqrt{(5-0)^2 + (2-0)^2}$

$$= \sqrt{25+4} = \sqrt{29} \text{ units}$$

Distance between origin and opposition Minister's Seat = $\sqrt{(-5-0)^2 + (7-0)^2}$

$$= \sqrt{25+49} = \sqrt{74} \text{ units}$$

SOLUTIONS (Sample Paper – 7)

1. **(b)** General from of a quadratic polynomial whose roots are α and β

$= x^2 - (\alpha + \beta)x + \alpha\beta$

Let $\alpha = (2+\sqrt{3})$ and $\beta = (2-\sqrt{3})$

$\therefore x^2 - (2+\sqrt{3}+2-\sqrt{3})x + (2+\sqrt{3})(2-\sqrt{3})$

$= x^2 - 4x + 4 - 3 = x^2 - 4x + 1$

2. **(c)** Let the HCF of numbers be x

then LCM of numbers be 2x.

Now, we know that product of two numbers = HCF × LCM

$\therefore \quad x \times 2x = 7200$

$\Rightarrow \quad 2x^2 = 7200 \Rightarrow x^2 = \frac{7200}{2} = 3600$

$\Rightarrow \quad x = \sqrt{3600} = 60$

$\therefore \quad$ HCF = 60 and LCM = 2x = 2 × 60 = 120

3. **(d)** For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1}{k} = \frac{-k}{-9} \Rightarrow k = \frac{9}{3} = 3$$

4. **(a)** In ΔABC and ΔADE,

DE || BC (Given)

$\angle ADE = \angle ABC$ (Corresponding angles)

$\angle AED = \angle ACB$ (Corresponding angles)

$\angle A = \angle A$ (Common)

$\therefore$ By AAA similarity criteria, $\Delta ABC \sim \Delta ADE$

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} \quad \text{[Corresponding sides of similar } \Delta\text{s]}$$

$$\therefore \quad \frac{BC}{DE} = \frac{AC}{AE} \Rightarrow \frac{8}{4} = \frac{AC}{3} \Rightarrow AC = 6 \text{ cm}$$

$\therefore \quad$ EC = AC – AE = 6 cm – 3 cm = 3 cm

5. **(d)** Discriminant for a quadratic equation = $b^2 - 4ac$

$$= \left[-(\sqrt{2}+1)\right]^2 - 4 \times 1 \times \sqrt{2}$$

$$= 2 + 1 + 2\sqrt{2} - 4\sqrt{2} \quad = 3 - 2\sqrt{2}$$

6. **(a)** Given, $\cos\theta = \frac{2}{3}$

$\therefore \quad \sec\theta = \frac{1}{\cos\theta} = \frac{3}{2}$ .....(i)

We know that $\sec^2\theta - \tan^2\theta = 1$

$\therefore \quad \tan^2\theta = \sec^2\theta - 1 = \left(\frac{3}{2}\right)^2 - 1$

$\Rightarrow \quad \tan^2\theta = \frac{9}{4} - 1 = \frac{9-4}{4} = \frac{5}{4}$ [From (i)]

7. **(b)** Given, 11 $\cot^2 A$ – 11 $\text{cosec}^2 A$

$\Rightarrow$ $-11 \times (\text{cosec}^2 A - \cot^2 A)$

$\Rightarrow$ $-11 \times 1 = -11$

8. **(a)** Coordinates of origin, O = (0, 0)

Given point is $(m \sin \theta, -m \cos \theta)$

$\therefore$ Distance between these two points = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(-m\cos\theta - 0)^2 + (m\sin\theta - 0)^2}$

$= \sqrt{m^2\cos^2\theta + m^2\sin^2\theta} = \sqrt{m^2(\cos^2\theta + \sin^2\theta)}$

Since, $\sin^2\theta + \cos^2\theta = 1$

$\therefore$ Distance between two points = $\sqrt{m^2}$ = m units

9. **(b)** Given, diameter = 42 cm $\Rightarrow$ radius = 21 cm

Let the radius of the hemispherical container be r cm

$\therefore$ Volume of the hemispherical container = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (21)^3$ = 19404 cm^3

= $\frac{19404}{1000} dm^3$ = 19.404 dm^3

10. **(d)** Given, $\Delta ABC \sim \Delta DEF$

$\therefore$ By CPST, $\angle A = \angle D = 50°$,

$\angle C = \angle F = 60°$

Now, in ΔDEF, $\angle D + \angle E + \angle F = 180°$

$\Rightarrow$ $50° + \angle E + 60° = 180°$

$\Rightarrow$ $\angle E = 180° - 110° = 70°$

11. **(d)** Given, $a_5 = -23$

$a + 4d = -23$

$a_{10} = -48 \Rightarrow a + 9d = -48$

Subtracting (i) from (ii), we get $5d = -25 \Rightarrow d = -5$

12. **(d)** According to the question, the smallest 2-digit prime number = 11

and the largest 2-digit composite number = 99

$\therefore$ HCF of 11 and 99 = 11

13. **(a)** We have, $(k - 2)x^2 - 8x + 16 = 0$

For equal roots, D = 0

$\Rightarrow (-8)^2 - 4(k - 2)(16) = 0$

$\Rightarrow$ $64 - 64k + 128 = 0$

$\Rightarrow$ $64k = 192 \Rightarrow k = 3$

14. **(a)** We have, $3x + 2y = 8$

and $3x - 2y = 4$

Adding (i) and (ii), we get

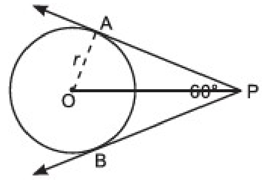
$6x = 12 \Rightarrow x = 2$

Substituting x = 2 in (i), we get

$3 \times 2 + 2y = 8 \Rightarrow 2y = 2 \Rightarrow y = 1$

15. **(c)** We have, $\angle APB = 60°$

As, tangents are equally inclined an angle on line segment joining external point and centre.



[Tangents drawn from an external point to a circle are equal]

So, $\angle APO = \angle BPO = 30°$

Now, $OA \perp PA$ [Tangent is $\perp$ to the radius]

In ΔOAP, $\tan 30° = \frac{OA}{PA} \Rightarrow \frac{1}{\sqrt{3}} = \frac{r}{PA}$

$\Rightarrow$ $PA = \sqrt{3}\ r$

So, $PA = PB = \sqrt{3}\ r$

16. **(b)** In ΔABC,

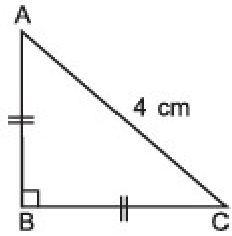
$AB^2 + BC^2 = AC^2$ (Pythagoras Theorem)

$\Rightarrow$ $2AB^2 = (4)^2$ $\{\because AB = BC\}$

$\Rightarrow$ $AB = 2\sqrt{2}$ cm = BC

$\therefore$ $ar(\Delta ABC) = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2}$ $= 4\ cm^2$



17. **(a)** Let $t_1 = 2x^2 + 3x - 2;\ t_2 = 2x^2 + 5kx + 6;\ t_3 = 2x^2 + 7x + 14$

Since t_1, t_2 and t_3 are in AP, then

$t_2 - t_1 = t_3 - t_2$

$\Rightarrow$ $2x^2 + 5kx + 6 - 2x^2 - 3x + 2 = 2x^2 + 7x + 14 - 2x^2 - 5kx - 6$

$\Rightarrow$ $10\ kx - 10x = 0$ $\Rightarrow$ $k = 1$

18. **(a)** Let S be the sample space then,

S = {HH, HT, TH, TT}; n(S) = 4

Let E be the event of getting at most 2 heads.

E = {HH, HT, TH, TT}, n(E) = 4

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{4}{4} = 1$

19. **(a)** Area of sector of circle = 385 cm^2

$\Rightarrow$ $\frac{\theta}{360°} \times \pi r^2 = 385$

$\Rightarrow$ $\frac{100°}{360°} \times \pi r^2 = 385$

$\Rightarrow$ $\pi r^2 = 1386$

So, area of the circle is 1386 cm^2

$\therefore$ Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. **(d)** The given system of equations is,

$x + 5y - 7 = 0$

$-15y - 3x + 5 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 1;\ b_1 = 5;\ c_1 = -7$$
$$a_2 = -3;\ b_2 = -15;\ c_2 = 5$$

Now, $$\frac{a_1}{a_2} = \frac{-1}{3};\ \frac{b_1}{b_2} = \frac{5}{-15} = \frac{-1}{3};\ \frac{c_1}{c_2} = \frac{-7}{5}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system of equations represents parallel lines

$\therefore$ Assertion (A) is false but reason (R) is true

21. Let the numerator be x and denominator be y.

So, required fraction = $\frac{x}{y}$

According to given conditions:

$$x + y = 12$$
$$y - x = 2$$

On adding (i) and (ii0, we get

$$2y = 14 \Rightarrow y = 7$$

Putting y = 7 in (i), we get x = 5

So, required fraction = $\frac{5}{7}$

22. LHS = $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$

$$= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A(1+\cos A)}$$

$$= \frac{2 + 2\cos A}{\sin A(1+\cos A)}$$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)} = 2 \text{ cosec } A = \text{RHS}$$

OR

LHS $= \frac{\sin A}{1-\cos A} + \frac{\tan A}{1+\cos A}$

$$= \frac{\sin A}{(1-\cos A)} + \frac{\sin A}{\cos A(1+\cos A)}$$

$$= \frac{\sin A \cos A + \sin A . \cos^2 A + \sin A - \sin A . \cos A}{(1-\cos A)(1+\cos A) . \cos A}$$

$$= \frac{\sin A(1+\cos^2 A)}{\cos A(1-\cos^2 A)} \quad = \frac{\sin A(1+\cos^2 A)}{\sin^2 A . \cos A}$$

$$= \frac{1}{\sin A . \cos A} + \frac{\cos^2 A}{\cos A . \sin A} \quad = \sec A . \text{cosec } A + \cot A = \text{RHS}$$

23. As, tangent drawn from an external point to a circle are equal.

So, CM = CL = 7 cm and BN = BL = 3 cm

Now, AN = AB – BN = 7 cm – 3 cm = 4 cm

$\therefore$ AM = AN = 4 cm

$\therefore$ AC = AM + CM = 4 cm + 7 cm = 11 cm

24. Perimeter of quadrant of circle = 19.5 cm

$\Rightarrow \quad r + r = \frac{\pi r}{2} = 19.5$

$\Rightarrow \quad 2r + \frac{11r}{7} = 19.5$

$\Rightarrow \quad \frac{25r}{7} = 19.5 \Rightarrow r = 5.46 \text{ cm}$

Now area of quadrant of a circle = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 5.46 \times 5.46 = 23.42 \text{ cm}^2$ (approx).

OR

As, angle in a semicircle is right angle, then

$\angle PRQ = 90°$

Now, $PQ^2 = PR^2 + QR^2$ (Pythagoras Theorem)

$\Rightarrow \quad PQ^2 = 6^2 + 8^2$

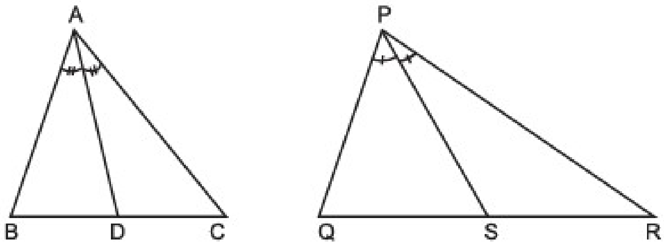
$\Rightarrow \quad PQ^2 = 100 \quad \Rightarrow \quad PQ = 10 \text{ cm}$

Perimeter of shaded region = length of arc PRQ + PR + QR

$= \frac{\pi d}{2} + 6 + 8$

$= (5\pi + 14)$ cm {$\because$ diameter (d) = PQ = 10 cm}

25. We have



$\Delta ABC \sim \Delta PQR$ (Given)

$\Rightarrow \quad \angle A = \angle P, \angle C = \angle R$ [Corresponding angles of similar Δ's are equal

As, AD bisects $\angle A$ and PS bisects $\angle P$, then

$\angle DAC = \frac{1}{2} \angle A$ and $\angle SPR = \frac{1}{2} \angle P$

Now, $\angle A = \angle P$

$\Rightarrow \quad \frac{1}{2} \angle A = \frac{1}{2} \angle P$

$\Rightarrow \quad \angle DAC = \angle SPR$ ….(i)

In ΔADC and ΔPSR,

$\angle DAC = \angle SPR$ [using (i)]

$\angle C = \angle R$ [$\because$ $\Delta ABC \sim \Delta PQR$]

$\Rightarrow \quad \Delta ADC \sim \Delta PSR$ (AA similarity)

$\Rightarrow \quad \frac{AD}{PS} = \frac{DC}{SR} = \frac{AC}{PR}$ [Corresponding sides of similar Δ's arc proportional]

$\Rightarrow \quad \frac{AD}{PS} = \frac{AC}{PR} \quad \Rightarrow \quad AD \times PR = AC \times PS$

26. Let us assume that $\sqrt{7}$ is a rational number.

So, $\sqrt{7} = \frac{p}{q}$ (when p and q are coprime integers and $q \neq 0$)

$\Rightarrow$ $p = \sqrt{7}\, q$

$\Rightarrow$ $p^2 = 7q^2$ …(i)

$\Rightarrow$ 7 divides p^2

$\Rightarrow$ 7 divides p …(ii)

$\Rightarrow$ p = 7r, when r is some integer

Putting p = 7r in (i), we get

$(7r)^2 = 7q^2$

$\Rightarrow$ $q^2 = 7r^2$

$\Rightarrow$ 7 divides q^2

$\Rightarrow$ 7 divides q …(iv)

From (ii) and (iii), we get 7 as the common factor of p and q. But this contradicts the fact the 'p' and 'q' are coprime.

Hence, our assumption is wrong

So, $\sqrt{7}$ is an irrational number.

27. Let $p(x) = x^2 - 7x + k$

Now, $\alpha + \beta = 7$ and $\alpha\beta = k$

$\alpha^2 + \beta^2 = 29$

$\Rightarrow$ $(\alpha + \beta)^2 - 2\alpha\beta = 29$

$\Rightarrow$ $(7)^2 - 2k = 29$

$\Rightarrow$ $49 - 2k = 29$

$\Rightarrow$ $2k = 20$

$\Rightarrow$ $k = 10$

28. $\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$

$= \frac{\cos\theta - 1 + \text{cosec}\,\theta}{\cot\theta + 1 - \text{cosec}\,\theta}$ [Dividing numerator and denominator by sin θ]

$= \frac{(\cos\theta + \cot\theta) - (\text{cosec}^2\theta - \cot^2\theta)}{(\cot\theta + 1 - \text{cosec}\,\theta)}$

$= \frac{(\text{cosec}\,\theta + \cot\theta) - (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta)}{(\cot\theta + 1 - \text{cosec}\,\theta)}$

$= \frac{(\text{cosec}\,\theta + \cot\theta)(1 - \text{cosec}\,\theta + \cot\theta)}{(\cot\theta + 1 - \text{cosec}\,\theta)}$ $= \text{cosec}\,\theta + \cot\theta$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1 + \cos\theta}{\sin\theta} \times \frac{1 - \cos\theta}{1 - \cos\theta} = \frac{1 - \cos^2\theta}{\sin\theta(1 - \cos\theta)}$

$= \frac{\sin^2\theta}{\sin\theta(1 - \cos\theta)} = \frac{\sin\theta}{1 - \cos\theta} = \text{RHS}$

29. We have

$\frac{x+1}{2} + \frac{y-1}{3} = 9 \Rightarrow 3x + 2y = 53$ …(i)

and $\frac{x-1}{3} + \frac{y+1}{2} = 8 \Rightarrow 2x + 3y = 47$ …(ii)

Adding (i) and (ii), we get

$$5x + 5y = 100 \Rightarrow x + y = 20 \quad \ldots(iii)$$

Subtracting (ii) from (i), we get

$$x/.,mn - y = 6 \quad \ldots.(iv)$$

Again, adding (iii) and (iv), we get c = 13

$\therefore$ From (iii), we get y = 7

$\therefore$ x = 13; y = 7 is the solution

OR

Let the digit at unit's and ten's place be x and y.

$\therefore$ original number = 10y + x

According to first condition,

$10y + x = 8(x + y) - 5$

$\Rightarrow \quad 7x - 2y = 5$

According to second condition,

$10y + x = 16(x - y) + 3$ or $10y + x = 16(y - x) + 3$

$\Rightarrow \quad 15x - 26y = -3 \quad \ldots(ii)$ or $17x - 6y = 3$

On solving (i) and (ii), we get

$$x = \frac{17}{19}, y = \frac{12}{19}$$

Because the number is two- digit not a fraction, so rejected x = $x = \frac{17}{19}$ and $y = \frac{12}{19}$

On solving (i) and (iii), we get

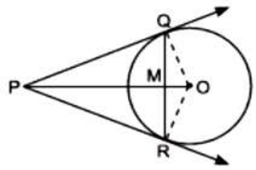
x = 3, y = 8

So, original number = 10 × 8 + 3 = 83

30. Join OQ and OR.

In ΔOQP and ΔORP,

PQ = PR	[Tangents drawn from an external point]
OP = OP	[Common]
OQ = OR	[Radii]
$\Rightarrow \Delta OQP \cong \Delta ORP$	(SSS)
$\Rightarrow \angle OPQ = \angle OPR$	(CPCT)



Suppose, OP intersects QR at M.

In ΔPMQ and ΔPMR,

PQ PR	[Tangents drawn from an external point]
$\angle QPM = \angle RPM$	[Using (i)]
PM = PM	[Common]
$\Rightarrow \Delta PMQ \cong \Delta PMR$	(SAS)
$\Rightarrow$ QM = RM and $\angle PMQ = \angle PMR$	(CPCT)
Now, $\angle PMQ + \angle PMR = 180°$	(Linear pair angles)
$\Rightarrow \angle PMQ = 90° = \angle PMR$	

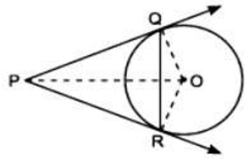
From (ii) and (iii), OP is the right bisector of QR

OR

Given: A circle with centre at O, to which tangents PQ and PR are drawn from an external point P.

To Prove: PQ = PR

Construction: Join OQ, OR and OP



Proof: As, tangent to a circle is always perpendicular to its radius at point of contact

So, $\angle OQP = \angle ORP = 90^\circ$

In ΔOQP and ORP,

$\angle OQP = \angle ORP$ (Each 90°)

OP = OP (Common)

OQ = OR (radii)

$\Rightarrow$ $\Delta OQP \cong \Delta ORP$ (RHS)

$\Rightarrow$ PQ = PR (CPCT)

31. Let S be the sample space when a coin is tossed 3 times.

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

$\therefore$ n(S) = 8

(i) Let E be the event of getting at least 1 tail.

E = {HHT, HTH, THH, TTT, TTH, THT, HTT}

$\therefore$ n(E) = 7

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

(ii) Let F be the event of getting exactly 2 tails.

F = {TTH, THT, HTT}

n(F) = 3

$$\therefore \quad P(E) = \frac{n(F)}{n(S)} = \frac{3}{8}$$

(iii) Let G be the event of getting atmost 2 tails.

G = {HHH, HHT, HTH, THH, TTH, THT, HTT}

n(G) = 7

$$\therefore \quad P(G) = \frac{n(G)}{n(S)} = \frac{7}{8}$$

32.

Class intervals	**Frequency**
0 – 15	10
15 – 30	7
30 – 45	m
45 – 60	15
60 – 75	n
75 – 90	12

As, $\sum f_i = 59 \quad \Rightarrow \quad 44 + m + n = 59$

$\Rightarrow$ m + n = 15 ...(i)

Since mode = 55, so modal class is 45 – 60

So, $f_1 = 15$, $f_0 = m$, $f_2 = n$, $l = 45$, $h = 15$

Mode = 55

$$\Rightarrow \quad l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 55$$

$\Rightarrow \qquad 45+\frac{15-m}{30-m-n}\times 15=55$

$\Rightarrow \qquad m - 2n = -15$ …(ii)

Solving (i) and (ii), we get

$m = 5; n = 10$

33. (i) **BPT:** If a line drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.

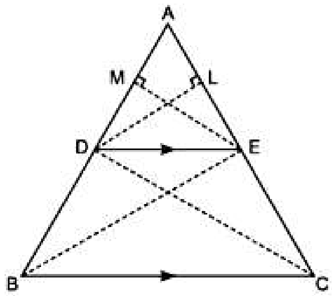
Given: ΔABC in which DE || BC

To Prove: $\frac{AD}{DB}=\frac{AE}{EC}$

Construction: Draw $DL \perp AC$ and $EM \perp AB$. Join BE and CD

Proof: ar(ΔADE) = $\frac{1}{2}\times AD\times EM$ ….(i)

and ar(ΔDBE) = $\frac{1}{2}\times DB\times EM$ …(ii)



Dividing (i) by (ii), we get

$$\frac{ar(\Delta ADE)}{ar(\Delta DBE)}=\frac{AD}{DB}$$ …(iii)

Also, $ar(\Delta ADE)=\frac{1}{2}\times AE\times DL$ …(iv)

and $ar(\Delta ECD)=\frac{1}{2}\times EC\times DL$ ….(v)

Dividing (iv) by (v), we get

$$\frac{ar(ADE)}{ar(ECD)}=\frac{AE}{EC}$$ …..(vi)

As, triangles ECD and DBE having same base and between same parallels are equal in areas.

So, ar(ΔECD) = ar(ΔDBE)

So, from (vi), we get

$$\frac{ar(\Delta ADE)}{ar(\Delta DBE)}=\frac{AE}{EC}$$ ….(vii)

From (iii) and (vi), we get

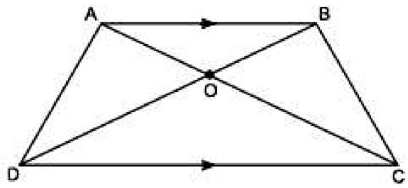
$$\frac{AD}{DE}=\frac{AE}{EC}$$

(ii) As, AB || DC and BD is transversal, then

$\angle ABO = \angle CDO$ [Alternate interior angles]

Again, AB || DC and AC is transversal, then

$\angle BAO = \angle DCO$ [Alternate interior angles]



In ΔAOB and ΔCOD,

$\angle ABO = \angle CDO$ [Proved above]

$\angle BAO = \angle DVO$ [Proved above]

$\Rightarrow \Delta AOB \sim \Delta COD$ (AA similarity)

$\Rightarrow \dfrac{AO}{OC} = \dfrac{OB}{OD} = \dfrac{AB}{CD}$ [Corresponding sides of similar Δ's are proportional

$\Rightarrow \dfrac{AO}{OC} = \dfrac{OB}{OD}$

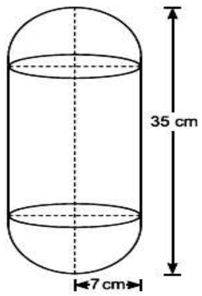
$\Rightarrow \dfrac{BO}{OD} = \dfrac{1}{2}$

34. We have,

base radius, r = 7 cm

Total height of solid = 35 cm

Now, height of cylindrical part, h = 35 cm – 14 cm = 21 cm



Volume of solid = $2 \times \dfrac{2}{3}\pi r^3 + \pi r^2 h$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 7 + \dfrac{22}{7} \times 7 \times 7 \times 21$

$= \dfrac{4312}{3} + 3234 = \dfrac{12014}{3} cm^3$

Outer CSA of solid = $2 \times 2\pi r^2 + 2\pi rh$

$= 4 \times \dfrac{22}{7} \times 49 + 2 \times \dfrac{22}{7} \times 7 \times 21 = 1540 \ cm^2$

OR

Base radius of cylinder, r = 6 cm

height of cylinder, h = 14 cm

Now, a right circular come of same height and same base as that of solid cylinder is removed from the cylinder

So, base radius of cone = r = 6 cm

height of cone = h = 14 cm

Volume of remaining solid = volume of cylinder – volume of cone

$= \pi r^2 h - \dfrac{1}{3}\pi r^2 h$

$$= \frac{2}{3}\pi r^2 h \qquad = \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14 \qquad = 1056 \text{ cm}^3$$

TSA of remaining solid = CSA of cylinder + CSA of cone + Area of circular base of cylinder

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + l + r)$

$$= \pi r\left(2h + \sqrt{r^2 + h^2} + r\right)$$

$$= \frac{22}{7} \times 6\left(2 \times 14 + \sqrt{6^2 + 14^2} + 6\right)$$

$$= \frac{132}{7}(34 + 15.23) \qquad = \frac{132}{7} \times 49.23 = 928.34 \text{ cm}^2 \text{ (approx.)}$$

35. The given quadratic equation is,

$3x^2 + kx - 7 = 0$

Since '1' is a root of the above equation, then it must satisfy it.

$3 \times (1)^2 + k \times 1 - 7 = 0 \quad \Rightarrow \quad k = 4$

Now, the other quadratic equation is,

$$p(x^2 + x) + \frac{k}{2} = 0$$

$\Rightarrow \; px^2 + px + 2 = 0$ [$\because$ k = 4]

Since the above equation has equal roots, then

$(p)^2 - 4 \times p \times 2 = 0 \quad \Rightarrow \quad p^2 - 8p = 0 \quad \Rightarrow \quad p(p - 8) = 0$

$\Rightarrow \quad p = 8$ or $p = 0$ (Rejected)

So, $p = 8$

OR

Let the usual speed be x km/h and increased speed be (x + 100) km/h

Distance to be travelled = 1500 km

Time taken to travel 1500 km with usual speed = $\left(\frac{1500}{x}\right)$hrs

Time taken to travel 1500 km with increased speed = $\left(\frac{1500}{x + 100}\right)$hrs

$\therefore$ According to question

$$\frac{1500}{x} - \frac{1500}{x + 100} = \frac{30}{60}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x + 100} = \frac{1}{3000}$$

$\Rightarrow \quad x^2 + 100x - 300000 = 0$

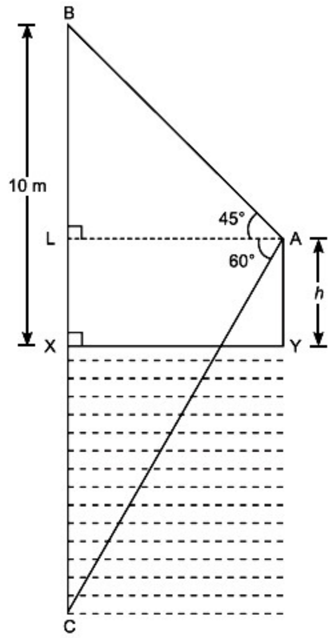
$\Rightarrow \quad x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow \quad (x - 500)(x + 600) = 0$

$\Rightarrow \quad x = 500$ or $x = -600$ (Rejected)

$\therefore$ Usual speed of the plane = 500 km/h

36. (i) As per the given information, the figure has been shown



(ii) In ΔBLA,

$\angle ABL + \angle ALB + \angle BAL = 180^\circ$

$\Rightarrow \angle B + 90^\circ + 45^\circ = 180^\circ$

$\Rightarrow \angle B = 45^\circ$

In ΔALC,

$\angle ACL + \angle ALC + \angle CAL = 180^\circ$

$\Rightarrow \angle C + 90^\circ + 60^\circ = 180^\circ$

$\Rightarrow \angle C = 30^\circ$

Now, $\sin^2 B + \cos^2 C = \sin^2 45^\circ + \cos^2 30^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2, \quad = \frac{1}{2} + \frac{3}{4} + \frac{5}{4}$$

(ii) As, $AL \perp BX$, then $LX = AY = h$

Now, $BL = BX - LX = (10 - h)m$

In ΔBLA, $\tan 45^\circ = \frac{BL}{AL}$

$\Rightarrow 1 = \frac{10 - h}{AL}$

$\Rightarrow AL = 10 - h$

In ΔCLA, $\tan 60^\circ = \frac{CL}{AL}$

$\Rightarrow \sqrt{3} = \frac{CX + LX}{10 - h}$

$\Rightarrow \sqrt{3} = \frac{BX + h}{10 - h}$ [$\because$ BX = CX]

$\Rightarrow \sqrt{3} = \frac{10 + h}{10 - h}$

$\Rightarrow h = 20 - 10\sqrt{3} = 20 - 10 \times 1.732 = 2.68$ m

OR

As, $BL = (10 - h)m = (10 - 2.68)m = 7.32$ m

In ΔALB, $\sin 45° = \frac{BL}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{7.32}{AB}$

$\Rightarrow$ AB = 7.32 × 1.414

$\Rightarrow$ AB = 10.35 m

37. The list of installments (in Rs) paid by Ravi every month is,

1000, 1200, 1400, ……….

The above list is in AP with first term, a = 1000 and common difference, d = 200

(i) amount paid in 30th installment = a_{30}

= a + 29d [$\because a_n = a + (n-1)d$]

= 1000 + 29 × 200

= 6800

$\therefore$ amount paid in 30th installment is Rs 6800

(ii) Amount paid in 30 installments = S_{30}

From formula, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{30} = \frac{30}{2}(2 \times 1000 + 29 \times 200) = 117000$

$\therefore$ Amount paid in 30 installments is Rs 117000

(iii) $S_n = 196000$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 196000$

$\Rightarrow \frac{n}{2}(2 \times 1000 + 200n - 200) = 196000$

$\Rightarrow n^2 + 9n - 1960 = 0$

$\Rightarrow (n - 40)(n + 49) = 0$

$\Rightarrow n = 40$ or $n = -49$ (rejected)

So, number of installment, n = 40

Now, $a_{40} = a + 39d = 1000 + 39 \times 200 = 8800$

$\therefore$ Amount paid in the last installment is Rs 8800

OR

$\frac{a}{a_{40}} = \frac{1000}{8800} = \frac{5}{44}$

So, the required ratio is 5: 44.

38. Distance between points (x_1, y_1) and (x_2, y_2) is,

distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) AB = $\sqrt{(6-3)^2 + (9-5)^2} = \sqrt{9+16} = 5\text{km}$

(ii) BC = $\sqrt{(12-6)^2 + (17-9)^2} = \sqrt{36+64} = 10\text{km}$

(iii) CD = $\sqrt{(15-12)^2 + (13-17)^2} = \sqrt{9+16} = 5\text{km}$

$\therefore$ Total distance travelled = AB + BC + CD

= 5 km + 10 km + 5 km = 20 km

OR

Let the coordinates of P be (α,β)

By section formula:

$$\alpha=\frac{2\times15+3\times3}{2+3}=\frac{39}{5}$$

$$\left[\text{using section formula } x=\frac{m_1x_2+m_2x_1}{m_1+m_2};\ y=\frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

$$\beta=\frac{2\times13+3\times5}{2+3}=\frac{41}{5}$$

$\therefore$ Coordinates of P are $\left(\frac{39}{5},\frac{41}{5}\right)$

SOLUTIONS (Sample Paper – 8)

1. **(a)** Since DE ||BC, so we have $\frac{AD}{AB} = \frac{AE}{AC}$

$\therefore \frac{3.4}{8.5} = \frac{AE}{13.5} \Rightarrow AE - \frac{3.4 \times 13.5}{8.5} \Rightarrow$ AE = 5.4cm

2. **(a)** We have cos $\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$ $(\because \cos 30° = \frac{\sqrt{3}}{2})$

Now, 3 tan θ – $\tan^3 \theta$ = 3 tan 30° – $\tan^3$ 30°

$= 3 \times \frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3 = \frac{3}{\sqrt{3}} - \frac{1}{3\sqrt{3}} = \frac{9-1}{3\sqrt{3}} = \frac{8}{3\sqrt{3}}$

3. **(c)** Since, p is prime

$\therefore$ p and p + 1 has no common factor other than 1.

$\therefore$ HCF of p and p + 1 = 1

& LCM of p and p + 1 = p × (p + 1) = p(p + 1)

4. **(d)** Let R(x, y) divides the line joining the points P(3,- 3) and Q(8, 4) in the ratio 2: 1 internally.

$\therefore \ x = \frac{2(8) + 1(3)}{2+1} = \frac{16+3}{3} = \frac{19}{3}$

and y $= \frac{2(4) + 1(-3)}{2+1} = \frac{8-3}{3} = \frac{5}{3}$

Thus, the coordinates of R are $\left(\frac{19}{3}, \frac{5}{3}\right)$

5. **(a)** Number of red balls = 4

Number of black balls = 6

Total number of balls in the bag = 4 + 6 =10

$\therefore$ P (getting a black ball) $= \frac{6}{10} = \frac{3}{5}$

6. **(a)** The given pair of equations is 9x + 3y + 5 = 0 and 6x + 2y + 7 = 0

Here, a_1 = 9, b_1 = 3, c_1 = 5, a_2 = 6, b_2 = 2, c_2 = 7

$\therefore \quad \frac{a_1}{a_2} = \frac{9}{6} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{3}{2}, \frac{c_1}{c_2} = \frac{5}{7}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ The given pair of equations represent parallel lines.

7. **(c)** We have, $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}}$

$= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{\sec^2\theta - 1}}$

$= \frac{2\sec\theta}{\sqrt{\tan^2\theta - 1}}$ $[\because \sec^2\theta - 1 = \tan^2\theta]$

$= \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\text{cosec}\,\theta$

8. **(d)** Total number of outcomes = 8
Favourable outcomes are (1, 2, 3, 4, 5, 6, 7, 8}
i.e., 8 in number.

$\therefore$ P(getting a number less than 9) $= \frac{8}{8} = 1$

9. **(a)** Given, 1 is the zero of the polynomial
$p(x) = ax^2 - 3(a-1)x - 1$
$\therefore$ $p(1) = 0$
$\Rightarrow$ $a(1)^2 - 3(a-1)(1) - 1 = 0$
$\Rightarrow$ $a - 3a + 3 - 1 = 0$
$\Rightarrow$ $-2a + 2 = 0 \Rightarrow a = 1$

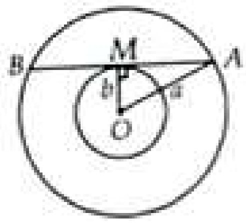
10. **(a)** ABCD is a rectangle having vertices B(4, 0), C(4,3) and D(0, 3)
Using distance formula,
$\sqrt{(4-0)^2 + (0-3)^2} = \sqrt{16+9} = 5$ units

11. **(c)** We have, $2a = a + (n-1)(b-a)$

$\Rightarrow$ $n - 1 = \frac{a}{b-a} \Rightarrow n = \frac{b}{b-a}$

$\therefore$ $S_n = \frac{b}{2(b-a)}(a+2a) = \frac{3ab}{2(b-a)}$

12. **(b)** Radius of larger circle = a
Radius of smaller circle = b



$\therefore$ In ΔOAM, we have
$OA^2 = OM^2 + AM^2$
$\Rightarrow$ $a^2 = b^2 + AM^2$
$\Rightarrow$ $AM^2 = a^2 - b^2$
$\Rightarrow$ $AM = \sqrt{a^2 - b^2}$
Now, length of chord of large circle is AB
$= 2AM = 2\sqrt{a^2 - b^2}$

13. **(c)** Here, n = 6, which is even

$\therefore$ Median = $\frac{1}{2}\left(\left(\frac{6}{2}\right)^{th} \text{term} + \left(\frac{6}{2}+1\right)^{th} \text{term}\right)$

$\Rightarrow 16 = \frac{1}{2}$ (3^{rd} term + 4^{th} term)

$\Rightarrow$ $32 = x - 2 + x$
$\Rightarrow$ $2x = 34 \Rightarrow x = 17$

14. **(a)** Total number of outcomes = 40
Multiples of 5 from 1 to 40 are {5, 10, 15, 20, 25, 30, 35, 40}
So number of favourable outcomes = 8

$\therefore$ Required probability = $\frac{8}{40} = \frac{1}{5}$

15. **(d)** All the values of (x, y) given in options (a), (b) and (c), satisfy the given pair of equations
$\therefore$ Option (d) is the correct answer.

16. **(d)** Let α and β be the zeroes of the polynomial
$p(x) = (p^2 - 23)x^2 - 2x - 12$

Then, $\alpha + \beta = -\frac{(-2)}{p^2 - 23} = \frac{2}{p^2 - 23}$

Also, sum of zeroes = $\alpha + \beta = 1$ [Given]

$\Rightarrow p^2 - 23 = 2 \Rightarrow p^2 = 25 \Rightarrow p = \pm 5$

17. **(b)** Since (x, y) is mid-point of line segment joining the point (3, 4) and (k, 7).

$\therefore \; x = \frac{3+k}{2}$ and $y = \frac{4+7}{2} = \frac{11}{2}$

Also, given 2x + 2y + 1 = 0 So, on putting values, we get

$3 + k + 11 + 1 = 0 \Rightarrow k + 15 = 0 \Rightarrow k = -15$

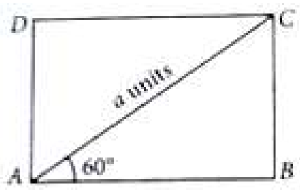
18. **(d)** In $\triangle ABC$, DE || BC

$\therefore \; \frac{AD}{AB} = \frac{AE}{AC}$ [By corollary of Thales theorem]

$\Rightarrow \frac{3}{7} = \frac{AE}{14} \Rightarrow AE = \frac{14 \times 3}{7} = 6$ cm

19. **(d)** In $\triangle ABC$, AC = a units, $\angle CAB = 60°$

$\therefore \sin 60° = \frac{BC}{AC} = \frac{BC}{a}$



$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{a} \Rightarrow BC = \frac{\sqrt{3}a}{2}$

Also, $\cos 60° \; \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{AB}{a}$

$\Rightarrow$ AB = a/2

$\therefore$ Area of rectangle ABCD = AB × BC

$= \frac{a}{2} \times \frac{\sqrt{3}a}{2} = \frac{\sqrt{3}a^2}{4}$

$\therefore$ Clearly, reason (R) is true statement

$\therefore$ Assertion (A) is false but reason (R) is true

20. **(c)** Clearly, reason (R) is false.

Now, the given equation is $kx^2 - 12x + 4 = 0$

If the roots are equals, then $b^2 - 4ac = 0$

$\Rightarrow (-12)^2 - 4(k)(4) = 0$

$\Rightarrow 144 - 16k = 0 \Rightarrow k = 144/16 = 9$

$\therefore$ Assertion (A) is true

21. Let the radius and the height of the cylinder be r and h respectively.

So, radius of the cone is r and height of the cone is 3h.

$\therefore$ Volume of the cylinder = $\pi r^2 h$

Volume of the cone = $\frac{1}{3}$ - $\pi r^2 (3h) = \pi r^2 h$

So, required ratio = $\frac{\pi r^2 h}{\pi r^2 h}$ = 1:1

22. **(a)** In $\triangle ABC$, DE || BC [Given]

$\therefore \; \frac{AD}{DB} = \frac{AE}{EC}$ [By Thales theorem]

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-2}$$

$\Rightarrow$ x(x – 1) = (x – 2)(x + 2) $\Rightarrow$ $x^2 - x = x^2 - 4 \Rightarrow x = 4$

OR

(b) Since, ΔABC is similar to ΔPQR.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \Rightarrow \frac{12}{9} = \frac{7}{x} = \frac{10}{y}$$

Taking first two terms, we get

$$\frac{12}{9} = \frac{7}{x} \Rightarrow x = \frac{9 \times 7}{12} = 5.25$$

Taking first and last terms, we get

$$\frac{12}{9} = \frac{10}{y} \Rightarrow y = \frac{9 \times 10}{12} = 7.5$$

23. Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OTP = 90^\circ$

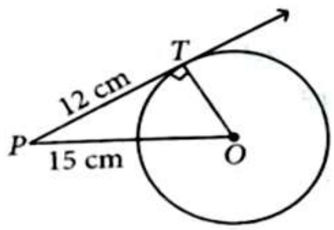
In right triangle OTP, we have

$OP^2 = OT^2 + PT^2$

$\Rightarrow 15^2 = OT^2 + 12^2$

$\Rightarrow OT^2 = 15^2 - 12^2 = 225 - 144 = 81$

$\Rightarrow OT = 9$



Hence, radius of the circle is 9 cm.

24. We have, $16x^2 - 8p^2x + (p^4 - q^4) = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

we get a = 16 b = $-8p^2$ and c = $(p^4 - q^4)$

$\therefore \quad b^2 - 4ac = (-8p^2)^2 - 4(16)(p^4 - q^4)$

$= 64p^4 - 64p^4 + 64q^4 = 64q^4 \geq 0$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8p^2) \pm \sqrt{64q^2}}{2(16)} = \frac{8p^2 \pm 8q^2}{32} = \frac{p^2 \pm q^2}{4}$$

Taking positive sign, $x = \frac{p^2 + q^2}{4}$

Taking negative sign, $x = \frac{p^2 - q^2}{4}$

25. **(a)** Given, 4tan θ = 3 $\Rightarrow$ tan θ = $\frac{3}{4}$

Also, $\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \left(\frac{3}{4}\right)^2} = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$

Now, $$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{4\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

[Dividing each term by cos θ]

$$= \frac{4\tan\theta - 1 + \sec\theta}{4\tan\theta + 1 - \sec\theta} \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{and} \sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{4\left(\frac{3}{4}\right) - 1 + \frac{5}{4}}{4\left(\frac{3}{4}\right) + 1 - \frac{5}{4}} = \frac{\frac{12}{4} - 1 + \frac{5}{4}}{\frac{12}{4} + 1 - \frac{5}{4}} = \frac{\frac{12-4+5}{4}}{\frac{12+4-5}{4}} = \frac{13}{11}$$

OR

(b) $\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1+\cos\theta+1-\cos\theta}{\sqrt{1-\cos\theta}\sqrt{1+\cos\theta}}$$

$$= \frac{2}{\sqrt{1-\cos^2\theta}} = \frac{2}{\sqrt{\sin^2\theta}} = \frac{2}{\sin\theta} = 2\text{cosec}\,\theta = \text{R.H.S}$$

26. The cumulative frequency table can be drawn as:

Class - interval	Frequency (f_i)	Cumulative frequency (c.f.)
0 – 10	2	2
10 – 20	5	7
20 – 30	x	7 + x
30 – 40	12	19 + x
40 – 50	17	36 + x
50 – 60	20	56 + x
60 – 70	y	56 + x + y
70 – 80	9	65 + x + y
80 – 90	7	72 + x + y
90 – 100	4	76 + x + y
Total	$\Sigma f_i = 76 + x + y$	

Given, $\Sigma f_i = 100$

$\Rightarrow$ 76 + x + y = 100 $\Rightarrow$ x + y = 24

Now, median = 52.5, which lies in the interval 50-60.

$\therefore$ Median class is 50-60.

So, l = 50, h = 10 f = 20, c.f $= 36 + x, \frac{n}{2} = \frac{100}{2} = 50$

$\therefore$ Median = $l + \left(\frac{\frac{n}{2} - \text{c.f.}}{f}\right) \times h$

$\Rightarrow$ 52.5 = 50 + $\frac{[50-(36+x)]}{20} \times 10 = 50 + \frac{14-x}{2}$

$\Rightarrow$ 52.5 – 50 = $\frac{14-x}{2}$ $\Rightarrow$ 2.5 × 2 = 14 – x

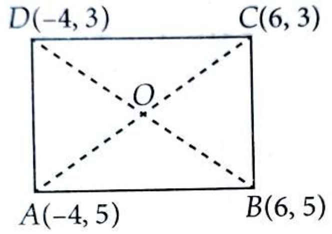
$\Rightarrow$ 5 = 14 – x $\Rightarrow$ x = 14 – 5 = 9

From (i) and (ii) we get 9 + y = 24

$\Rightarrow$ y = 24 – 9 = 15

$\therefore$ x = 9 and y = 15

27. **(a)** We have ABCD is a rectangle, where AC and BD are its diagonals



Now, $AC = \sqrt{[6-(-4)]^2+(3-5)^2}$

$= \sqrt{(10)^2+(-2)^2} \qquad = \sqrt{100+4} = \sqrt{104}$ units

and $BD = \sqrt{(-4-6)^2+(3-5)^2} = \sqrt{(-10)^2+(-2)^2}$

$= \sqrt{100+4} = \sqrt{104}$ units

$\Rightarrow$ AC = BD

Hence, diagonals of rectangle ABCD are equal.

Let O is the mid-point of both AC and BD

Using mid-point formula, we have

coordinates of 0 from AC are $\left(\frac{6+(-4)}{2}, \frac{3+5}{3}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

Coordinates of O from BD are $\left(\frac{-4+6}{2}, \frac{3+5}{2}\right)$

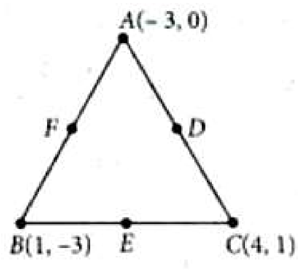
$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

Thus, AC and BD bisect each other at O

OR

(b) Let D, E and F be the mid-points of the sides AC, BC and AB respectively.

Then, the coordinates of D are



$\left(\frac{-3+4}{2}, \frac{0+1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

Coordinates of E are $\left(\frac{1+4}{2}, \frac{-3+1}{2}\right) = \left(\frac{5}{2}, \frac{-2}{2}\right) = \left(\frac{5}{2}, -1\right)$

Coordinates of F are $\left(\frac{-3+1}{2}, \frac{0-3}{2}\right) = \left(\frac{-2}{2}, \frac{-3}{2}\right) = \left(-1, \frac{-3}{2}\right)$

Using distance formula, lengths of medians are

$AE = \sqrt{\left[\frac{5}{2}-(-3)\right]^2+(-1-0)^2}$

$= \sqrt{\left(\frac{11}{2}\right)^2 + 1} = \sqrt{\frac{121}{4} + 1} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$ units

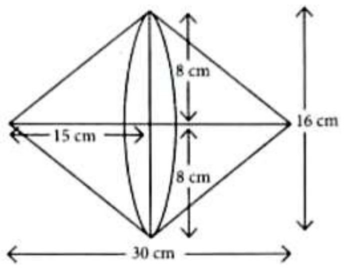
$BD = \sqrt{\left(\frac{1}{2} - 1\right)^2 + \left[\frac{1}{2} - (-3)\right]^2} = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{\sqrt{50}}{2}$ units

$CF = \sqrt{(-1-4)^2 + \left(\frac{-3}{2} - 1\right)^2}$

$= \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100+25}{4}} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$ units

28. If two cones with same base and height are joined together along their bases, then the shape so formed looks as the figure given below:



Given that diameter of cone (d) = 16cm
so radius of cone (r) = 8cm
and height of cone (h) = 15cm

$\therefore$ Surface area of the shape so formed = 2 x curved surface area of the shape so formed [$\because$ Both cones are identical]

$= 2 \times \pi r l = 2 \times \pi \times r \times \sqrt{h^2 + r^2}$

$= 2 \times \frac{22}{7} \times 8 \times \sqrt{8^2 + 15^2} = 2 \times \frac{22}{7} \times 8 \times 17\ cm^2$

Surface area $\frac{5984}{7}$ cm^2 = 855 cm^2 (approx.)

29. Given, $\frac{3+5+7+\text{...to n terms}}{5+8+11+\text{...to10 terms}} = 7$

$\Rightarrow \frac{\frac{n}{2}\{2(3)+(n-1)2\}}{\frac{10}{2}\{2(5)+(10-1)3\}} = 7$

$\Rightarrow \frac{\frac{n}{2}\{2(3)+(n-1)2\}}{\frac{10}{2}\{2(5)+(10-1)3\}} = 7$

$\Rightarrow 2n^2 + 4n - 2590 = 0 \Rightarrow n^2 + 2n - 1295 = 0$

$\Rightarrow (n + 37)(n - 35) = 0 \Rightarrow n = 35$ or $n = -37$

$\therefore$ n = 35 ($\because$ Number of terms cannot be negative.)

30. **(a)** Given, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle D = \angle E$ ...(i)

We have to prove that ΔABC is an isosceles triangle.

Now, in ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$ [Given]

$\therefore$ DE ||BC [By Converse of Basic Proportionality Theorem]

Also, $\angle D = \angle B$ and $\angle E = \angle C$ [Corresponding angles] ...(ii)

From (i) and (ii), we get $\angle B = \angle C \Rightarrow AB = AC$

[Sides opposite to equal angles are equal]

$\therefore$ ΔABC is an isosceles triangle

OR

(b) In ΔSPR, PQ || RB

$\Rightarrow \frac{SP}{SB} = \frac{SQ}{SR}$ [By B.P.T.] …(i)

Also, in ΔSPR, PR || QC

$\Rightarrow \frac{SC}{SP} = \frac{SQ}{SR}$ [By B.P.T.] …(ii)

From (i) and (ii), we get $\frac{SP}{SB} = \frac{SC}{SP} \Rightarrow SP^2 = SB \times SC$

Hence proved.

31. Let OP be the tower of height h m and A,B be two cars such that AB = 100m.

Now in ΔOPA, $\tan 60^\circ = \frac{OP}{OA}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ……(i)

In ΔOPB, $\tan 45^\circ = \frac{OP}{OB}$

$\Rightarrow 1 = \frac{h}{x+100}$

$\Rightarrow x + 100 = h \Rightarrow h - x = 100$ ….(ii)

$\Rightarrow h - \frac{h}{\sqrt{3}} = 100$ [From (i) and (ii)]

$\Rightarrow \frac{(\sqrt{3}-1)}{\sqrt{3}} h = 100 \quad \Rightarrow \quad h = \frac{100\sqrt{3}}{\sqrt{3}-1} = \frac{100\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{100(3+\sqrt{3})}{2} = 50 \times (3 + 1.73) = 236.5\,\text{m}$

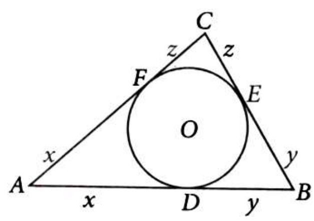
Therefore, the height of the tower is 236.5 m.

32. Since, tangents drawn from an external point to a circle are equal.

$\therefore$ AD = AF = x (say)

BD = BE = y (say)

CE = CF = z (say)



According to the question,

AB = x + y = 24 cm ….(i)

BC = y + z = 16 cm ….(ii)

AC = x + z = 20 cm ….(iii)

On subtracting (iii) from (i), we get

y – z = 4 .....(iv)

Adding (ii) and (iv), we get

2y = 20 ⇒ y = 10 cm

Substituting the value of y in (i) and (ii), we get

x = 14 cm, z = 6 cm

∴ AD = 14 cm, BE = 10 cm and CF = 6 cm.

33. (a) Let cost of one pencil be Rs. x and that of one chocolate be Rs. y.

According to the condition-I, we have

2x + 3y = 1

Also, according to the condition –II, we have

x + 2y = 7

Thus, algebraic representation of given situation is :

2x + 3y = 11 ....(i)

and x + 2y = 7 ....(ii)

For geometrical representation of given problem, we find two solutions of given system of linear equations represented by (i) and (ii).

From (i), $y = \frac{11-2x}{3}$

Table of solutions for (i) is :

x	1	4
y	3	1

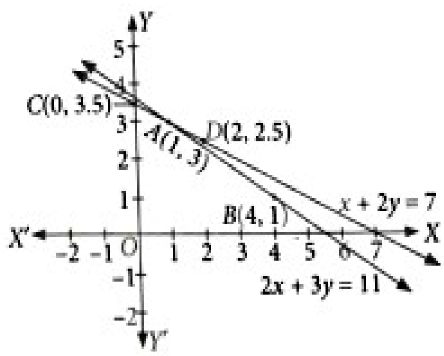
From (ii), $y = \frac{7-x}{2}$

Table of solutions for (ii) is :

x	0	2
y	3.5	2.5

Now, plotting the points A(1, 3) and B(4, 1) on graph paper and joining them, we get the line 2x + 3y = 11.

Similarly, plotting the points C(0, 3.5) and D(2, 2.5) on graph paper and joining them, we get the line x + 2y = 7.



From the graph, we see that the two lines are intersecting each other at point A(1, 3).

OR

(b) Graph system of linear equation is

x – y = 1 ⇒ y = x – 1 ....(i)

2x + y = 8 ⇒ y = 8 – 2x ....(ii)

Table of solutions for (i) is :

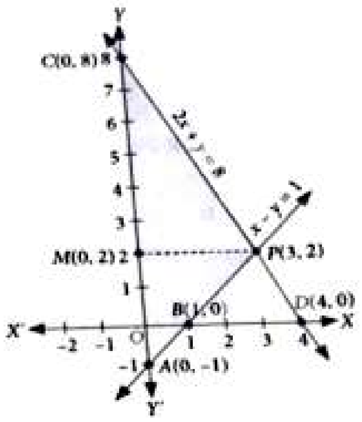
x	0	1
y	-1	0

Table of solutions for (ii) is :

x	0	4
y	8	0

Plot the points A(0, -1), B(1, 0) on a graph paper and join them to get the line AB.

Similarly, plot the points C(0, 8), D(4, 0) on the same graph paper and join them to get the line CD.



Clearly, the two lines intersect at P(3, 2).

The co-ordinates of area enclosed by the lines represented by the given equations and the y-axis are P(3, 2), A(0, -1) and C(0, 8).

Now, required area = Area of the shaded region

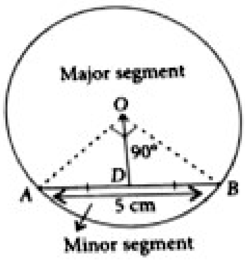
= Area of ΔPAC = $\frac{1}{2}(\text{Base} \times \text{Height})$

$\frac{1}{2}(\text{AC} \times \text{PM}) = \frac{1}{2}(9 \times 3) = 13.5\text{sq.units}$

34. (a) Let the radius of the circle be r.

Given that, length of chord of a circle, AB = 5 cm

and central angle of the sector AOBA (θ) = 90^o



Now, in right ΔAOB,

$(AB)^2 = (OA)^2 + (OB)^2$ [By Pythagoras theorem]

$\Rightarrow (5)^2 = r^2 + r^2 \Rightarrow 2r^2 = 25$

$\Rightarrow r = \frac{5}{\sqrt{2}}\text{cm}$

$\therefore$ Area of ΔAOB = $\frac{1}{2} \times OB \times OA$

$= \frac{1}{2} \times \frac{5}{\sqrt{2}} \times \frac{5}{\sqrt{2}} = \frac{25}{4}\text{cm}^2$

Now, area of sector AOBA

$$= \frac{\theta \times \pi r^2}{360^o} = \frac{90^o \times \left(\frac{5}{\sqrt{2}}\right)^2 \times \pi}{360^o} = \frac{25 \times \pi}{2 \times 4} = \frac{25\pi}{8}\text{cm}^2$$

$\therefore$ Area of minor segment = Area of sector AOBA – Area of ΔAOB = $\left(\frac{25\pi}{8} - \frac{25}{4}\right)\text{cm}^2$

Now area of the circle = πr^2 = $\pi\left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25\pi}{2}\text{cm}^2$

$\therefore$ Area of major segment = Area of circle – Area of minor segment

$= \frac{25\pi}{2} - \left(\frac{25\pi}{8} - \frac{25}{4}\right) = \frac{25\pi}{8}(4-1) + \frac{25}{4}$

$= \left(\frac{75\pi}{8} + \frac{25}{4}\right)cm^2$

$\therefore$ Difference of the areas of two segments of a circle

= Area of major segment – Area of minor segment

$= \left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{8} - \frac{25}{4}\right) = \left(\frac{75\pi}{8} - \frac{25\pi}{8}\right) + \left(\frac{25}{4} + \frac{25}{4}\right)$

$= \frac{75\pi - 25\pi}{8} + \frac{50}{4} = \frac{50\pi}{8} + \frac{50}{4} = \left(\frac{25\pi}{4} + \frac{25}{2}\right)cm^2$

Hence, the required difference of the areas of two segments is $\left(\frac{25\pi}{4} + \frac{25}{2}\right)cm^2$.

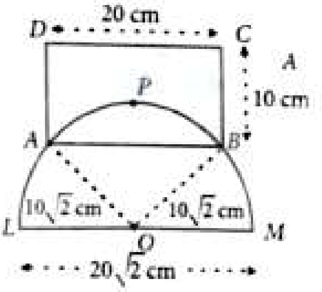
OR

(b) In ΔOAB, OA = OB = $10\sqrt{2}$cm (radius)

and AB = 20 cm (Given as the side of the rectangle)

$\Rightarrow AB^2 = OA^2 + OB^2 \Rightarrow \angle AOB = 90^o$

We take a point P on the arc AB of the semi-circle.



Now, are of the segment APB = The area of the sector OAPB – area of the ΔOAB

$= \left\{\frac{90^o}{360^o} \times \pi \times (10\sqrt{2})^2 - \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}\right\}cm^2$

$= \left(\frac{1}{4} \times \pi \times 200 - \frac{1}{2} \times 200\right)cm^2 = \left(\frac{\pi}{2} - 1\right) \times 100cm^2$

Area of the shaded region = Area of the rectangle ABCD – area of the segment APB

$= 20 \times 10cm^2 - \left(\frac{\pi}{2} - 1\right) \times 100\,cm^2$

$= \left\{200 - \frac{\pi}{2} \times 100 + 100\right\}cm^2 = \left(300 - \frac{\pi}{2} \times 100\right)cm^2$

$= \left(3 - \frac{22}{14}\right) \times 100cm^2 = 142.85cm^2$

35. Let us assume $\sqrt{7}$ be a rational number.

So, we can find integers a and b ($b \neq 0$) such that $\sqrt{7} = \frac{a}{b}$,

where a and b ($\neq 0$) are co-prime positive integers.

So, $\sqrt{7}b = a$ …..(i)

Squaring both sides, we get

$7b^2 = a^2$. Therefore, 7 divides a^2.

$\Rightarrow$ 7 divides a. ….(ii)

So, we can write a = 7c, for some integer c.

Putting a = 7c in (i), we get $\sqrt{7}b = 7c$

$\Rightarrow$ $7b^2 = 49c^2 \Rightarrow b^2 = 7c^2$ [Squaring both sides]

$\therefore$ 7 divides $b^2 \Rightarrow$ 7 divides b. ….(iii)

From (ii) and (iii), a and b have 7 as a common factor. But this contradicts the fact that a and b are c-primes.

$\therefore$ Our assumption that $\sqrt{7}$ is rational, is wrong.

Hence, $\sqrt{7}$ is irrational.

36. (i) Thief is 100 m ahead of policeman.

(ii) Thief is running at uniform speed of 100 m/min.

$\therefore$ In 5 minutes distance covered = 5 × 100m + 100m = 600 m

So, distance covered by thief is 600 m.

(iii) For policeman, distance covered in 5 mintues

$$= \frac{5}{2}[2 \times 100 + (5-1) \times 1] \qquad [\because S_n = \frac{n}{2}\{2a + (n-1)d\}]$$

$$= \frac{5}{2}(200 + 4) = 510m$$

OR

Required distance = $100 \times 4 - \frac{3}{2}[2 \times 100 + (3-1) \times 1]$

= 400m – 3 × 101m = 400m – 303m = 97 m

37. (i) Number of chocolates = x

Number of pancakes = y

$\therefore$ x + y = 400 ….(1)

A.T.Q., y + 50 = 4(x – 50)

$\Rightarrow$ 4x – 200 = y + 50

$\Rightarrow$ 4x – y = 250 …..(2)

(1) and (2) represent the situations.

(ii) We have, x + y = 400 and 4x – y = 250

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{4}, \frac{b_1}{b_2} = -1, \quad \frac{c_1}{c_2} = \frac{400}{250} = \frac{8}{5}$$

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the given system of equation has unique solution.

(iii) (a) Adding (1) & (2), we get 5x = 650 $\Rightarrow$ x = 130

Thus, Shweta had 130 chocolates.

From (1), x + y = 400

$\Rightarrow$ y = 400 – 130 = 270

Thus, Shweta had 270 pancakes.

OR

(iii) (b) Cost of 130 chocolates = Rs. 20 × 130 = Rs. 2600

Cost of 270 pancakes = Rs. (3 × 270) = Rs. 810

$\therefore$ Total cost of chocolates & pancakes = Rs. (2600 + 810) = Rs. 3410

38. (i) Total number of chocolates

$= 20 + 25 + 33 + 42 + 70 = 200$

Number of chocolates of brand E = 70

$\therefore$ P(chocolate is of brand E) = $\frac{70}{200} = \frac{7}{20}$

(ii) Total number of chocolates = 200

Number of chocolates of brand B = 35

$\therefore$ P(chocolate is brand B) i.e., P(B) = $\frac{35}{200}$

Thus, the probability of the chocolate is not of brand

$B = 1 - P(B) = 1 - \frac{35}{200} = \frac{200-35}{200} = \frac{165}{200} = \frac{33}{40}$

(iii) (a) Number of chocolates of brand C = 33

$\therefore$ P(chocolate is of brand C) = $\frac{33}{200}$

OR

(iii) (b) Number of chocolates of brand A = 20 and number of chocolates of brand D = 42

$\therefore$ Total number of chocolates of brand A and D outcomes = 20 + 42 = 62

$\therefore$ P(chocolates is of brand D or brand A) = $\frac{62}{200} = 0.31$